

A Fuzzy Inference System for Credit Scoring using Boolean Consistent Fuzzy Logic

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Abstract

This study proposes implementation of Boolean consistent fuzzy inference system for credit scoring purposes. Fuzzy inference system (FIS) allows domain experts to express their knowledge in the form of fuzzy rules, which enables combination of automatic rating with human judgment. Crucial for this model is that fuzzy rules are being evaluated using Boolean consistent fuzzy logic, which preserves all Boolean axioms. Experimental results show that the Boolean consistent FIS outperforms the conventional FIS in terms of classification accuracy, precision, and recall. Consistent fuzzy logic could contribute to the rightful approval of more loans which in turn would have positive effects on economic growth.

Keywords: Fuzzy Inference System; Boolean Consistent Fuzzy Logic; Banks; Credit Scoring; Performance.

1. Introduction

Micro, small, and medium enterprises (MSME) in developed and emerging economies represent an

important sector, which needs to be vigorous in order to be able to generate economic development. This sector is essential for creating employment, increasing international trade, and establishing an entrepreneurship spirit. However, the banking sector is hesitant to lend to

MSM sector due to their perception that this sector is connected to high risk. Nevertheless, there is some evidence that banks lend more freely to this sector in the form of unsecured loans when credit scoring models are used. Granting loans to MSM sector is especially important because this sector has limited access to alternative financing sources like venture capital or private equity.^{1,2}

Loans represent the largest source of credit risk for the banks, where special emphasis can be put on the MSME loans and banks' lending procedure for this sector. "Banks should develop and utilize internal risk rating systems in managing credit risk. The rating system should be consistent with the nature, size and complexity of a bank's activities".³

Every banking/lending institution tailors credit process to its own needs. Credit process begins with credit application which contains borrowers' historical information, and which is used in credit risk measurement. The result of a credit analysis is assigned internal risk rating/score which enumerates borrowers' probability of default. Credit risk approval is based on this analysis.

The credit worthiness of an enterprise is usually determined by its internal characteristics and ability to respond to external influences. The credit risk of an enterprise can be derived from financial risk, business risk, industry risk, and its management style. Credit analysis uses ratio analysis to identify risks and potential growth prospects. Lenders may use different ratios, which can be calculated according to their preferences and sorted into a few categories: profitability, performance, liquidity, solvency, efficiency, and debt.⁴ However, there is no correct value for a ratio. Yet, analyst experience and judgment will be required to assess whether the value is too low, appropriate or too high. There is no single analysis that would be effective for all types of loans, but data analysis should focus on assessment of the enterprises risks and repayment ability, and implemented criteria would vary depending on the counterpart.^{5,4} Internal rating systems require business rules for their implementation.⁵ In order for a lending institution to promote lending to MSME it is encouraged to use automated credit scoring models for credit analysis.

Credit scoring can be performed using various methods (see Ref. 6-20). Statistical methods were first one to be implemented, such as probit, logit, and

discriminant analysis. According to the reviewed literature neural networks, support vector machines, fuzzy inference systems, neuro-fuzzy systems, and various hybrid models were tested for credit scoring. Recent studies have shown that artificial intelligence methods achieved better performance than traditional statistical methods.²¹

Therefore, in this research we will rely on the fuzzy system and propose a new fuzzy inference system for credit score determination, based on the consistent fuzzy logic. More precisely, we propose new fuzzy inference system based on expert rules which include financial ratios, and are based on the premises of the credit analysis. These rules are going to be evaluated using Boolean consistent fuzzy logic. The motivation for this application of Boolean consistent fuzzy logic is that it guarantees that all Boolean axioms are valid on the whole interval $[0, 1]$ not only on the set $\{0, 1\}$. As a consequence, Boolean consistent fuzzy logic will produce different results comparing to conventional fuzzy logic.

Rest of the paper is structured as follows, in part 2 detailed review of literature is presented; part 3 is related to the theoretical background of the fuzzy reasoning; in part 4 a new fuzzy inference system is developed, and part 5 concludes the paper.

2. Literature Review

Credit scoring represents an important financial concern, especially after the credit crunch. It was widely explored in the past using different methods.

Altman²² developed Z-score, which was widely used to predict bankruptcy. Also, Myers and Forgy²³ performed several discriminant and multiple regression analyses. Gardner and Mills²⁴ use the logit regression model to estimate the probability of default, while Jacobson and Roszbach¹⁰ applied a bivariate probit model. Furthermore, Henley and Hand²⁵ used k-Nearest neighbour algorithm. Servigny and Renault²⁶ anticipate that in the near future banks will move towards integration of nonparametric techniques and machine learning models.

Numerous authors implemented neural networks for solving the problem of credit scoring. Zhao et al.²⁰ examine the effects of training, number of hidden units and optimize the data distribution in datasets used for training. Accuracy of the model is increasing with the

rising number of hidden units. Artificial neural networks with different training algorithms, inspired by the neurons' biological property of metaplasticity are studied in Marcano-Cedeno et al.²⁷. Alejo et al.²⁸ used multilayer perceptron neural network and proved that cost functions have an influence on the training process. West¹⁸ investigates the accuracy of five neural network models (multilayer perceptron, mixture-of-experts, radial basis function, learning vector quantization and fuzzy adaptive resonance) benchmarked against traditional methods. Khashman²⁹ implements supervised neural network models. Generally neural networks showed that they are a valid model for credit scoring classification. Bayesian network model performs better than logistic regression and neural network models if we look at the accuracy, sensitivity, precision and the receiver characteristic curve.³⁰

Neuro-fuzzy systems were also used as a model for credit score classification. Fang³¹ and Piramuthu³² used neuro-fuzzy systems and neural networks, where ANFIS proved to be more accurate. Malhotra and Malhotra³³ showed that the neuro-fuzzy system performs better than discriminant analysis.

Several hybrid models are also used. Lee et al.¹⁴ integrated neural networks with discriminant analysis. They used the result of discriminant analysis to simplify the backpropagation neural network structure. Another approach, based on the different feature selection algorithms and classification algorithms is used in the paper by Koutanaei et al.¹² Ala'rajand and Abbod⁶ tested the ability of linear regression combined with multivariate adaptive regression splines in order to improve predictions given by neural networks, support vector machines, random forest, decision trees, and naive Bayes. The results show that proposed approach which includes two preprocessing models improved prediction performance against base classifiers. Xiao et al.¹⁹ ensemble classification approach based on supervised clustering, and improves accuracy of credit scoring of individual customers. Chen and Cheng³⁴ and Bellalah et al.³⁵ also proposed different hybrid models, for the credit rating of banks and default probabilities of corporations, respectively. For bank credit failure hybrid ensemble learning models can be used with high predictive power.³⁶

Many papers analyze the performance and compare the results obtained by different models, discussing advantages and disadvantages of each method. Jo et

al.³⁷, Sabzevari et al.¹⁷, Ince and Aktan⁹, Desai et al.⁷, and Tsengand and Hu³⁸ showed that for credit scoring or bad loan classification, data mining models provide better results than statistical techniques. Kao et al.¹¹ used Bayesian latent variable model and compared it with the performance of discriminant analysis, logistic regression, neural networks, multivariate adaptive regression splines, and support vector machine for the assessment of the potential credit card holders. Proposed model increases accuracy and reduces type I error. Support vector machines and neural networks were used by Huang et al.²¹ Study by Harris⁸ compares clustered support vector machine with other nonlinear support vector machine based techniques and found that clustered support vector machine to achieve acceptable results in the classification of individuals' creditworthiness. Lee et al.¹⁵ compared and analyzed results given by classification and regression tree, multivariate adaptive regression splines, discriminant analysis, logistic regression, neural networks, and the support network machine. Results show that classification and regression tree and multivariate adaptive regression splines outperform other methods in credit scoring classification of customers.

Based on the previous researches, we can see that artificial intelligence methods are extensively implemented related to credit risk. Generally, researches show that methods of artificial intelligence prove to be suitable for credit risk problems since they outperform statistical methods. Especially, neural networks are widely used for credit risk evaluation. The drawback of neural networks is its black box system and they do not presume any knowledge about the system. At the other hand, fuzzy systems can incorporate expert knowledge regarding the problem. Therefore, in this research we will rely on the fuzzy system and propose a new fuzzy inference system for credit score determination.

3. Fuzzy Modeling using Consistent Fuzzy Logic

Fuzzy modeling is widely used in various domains because it enables a system to be modeled based on its linguistic description. Fuzzy modeling can be performed using conventional and consistent fuzzy reasoning, among others. Consistent fuzzy logic satisfies all Boolean axioms and it is therefore, the preferred approach in this paper for evaluation of fuzzy rules. We

will present both types of reasoning, but we shall put emphasis on the consistent fuzzy logic.

3.1. Fuzzy logic and fuzzy inference system

Classical set theory only answers the question whether an element meets the characteristics, i.e. belongs to the given set (marked with 1) or does not belong to the given set (marked with 0). Fuzzy logic presents an extension of binary values 0 and 1 to the whole interval $[0, 1]$, which enables us to refine the degree of association of an element to the given set.³⁹ Among other benefits, fuzzy logic proved to be adequate when working with linguistic variables, i.e. variables whose values are words or sentences in a natural language.⁴⁰ Linguistic variables are mapped to the unit interval based on a membership function.

One of the most important issues of fuzzy logic is fuzzy inference system. Fuzzy inference systems are fuzzy logic based support systems that model input output relationships. Relations between input and output variables are defined by fuzzy rules.

3.2. Boolean consistent fuzzy logic

In the proposed inference system, fuzzy rules will be evaluated using Boolean consistent fuzzy logic proposed by Radojevic.^{41, 42, 43} The reason for such an application lies in the fact that the conventional fuzzy set theory is not in the Boolean frame.⁴² More precisely, the conventional fuzzy set theory does not satisfy the axioms of excluded middle and contradiction. The excluded middle and contradiction axioms declare, respectively Eq. (1) and (2):

$$\mu_A(x) \vee \overline{\mu_A(x)} = 1 \quad (1)$$

$$\mu_A(x) \wedge \overline{\mu_A(x)} = 0 \quad (2)$$

where: $\mu_A(x)$ is a membership function of x in A .

For illustrating purposes that Boolean axioms are not satisfied, we put the values into the equations (1) and (2). For example, if $\mu_A(x) = 0.45$ that would imply that negation of property is $\mu_A(x) = 0.55$.

For the max or algebraic sum selected for the fuzzy disjunction, we get:

$$\mu_A(x) \vee \overline{\mu_A(x)} = \max(0.45, 0.55) = 0.55$$

$$\begin{aligned} \mu_A(x) \vee \overline{\mu_A(x)} &= \mu_A(x) + \overline{\mu_A(x)} - \mu_A(x) \cdot \overline{\mu_A(x)} \\ &= 0.45 + 0.55 - 0.45 \cdot 0.55 = 0.7525 \end{aligned}$$

Similarly, for the min or product selected for the fuzzy conjunction, we get:

$$\mu_A(x) \wedge \overline{\mu_A(x)} = \min(0.45, 0.55) = 0.45$$

$$\mu_A(x) \wedge \overline{\mu_A(x)} = \mu_A(x) \cdot \overline{\mu_A(x)} = 0.45 \cdot 0.55 = 0.2475$$

Therefore, the application of the Interpolative Boolean algebra (IBA) is proposed. Radojevic⁴² defined that IBA is an illustrative name of the real-valued realization of Boolean algebra. Boolean logic is just a special case of IBA, because its' laws are valid on the whole interval $[0, 1]$ not only $\{0, 1\}$. IBA is an atomic algebra because it has a finite number of elements. Algebraic structure of IBA is set of $\langle BA(\Omega), \cap, \cup, \neg \rangle$, where $BA(\Omega)$ is the set of all the possible elements generalized by primary elements from Ω ⁴³. Primary elements $\{a_1, a_2, \dots, a_n\}$ from Ω have a property that any of them cannot be expressed by other primary elements. IBA distinguishes two levels: structural level and value level. Structural level requires that set of rigorous predefined structural transformations must be executed before we go to the value level. On the value level, values will be imputed in the final structure and all Boolean axioms will hold. Hence, partial order at the value level is persevered. Any logical function can be uniquely transformed into a corresponding generalized Boolean polynomial (GBP) using IBA. Steps for transformation of operations of union, intersection and negation are given in Eq. (3)^{41, 42} where ϕ and ψ are two different IBA elements:

$$\begin{aligned} \phi, \psi &\in BA(\Omega) \\ (\phi \cap \psi) &= (\phi)^\otimes \otimes (\psi)^\otimes \\ (\phi \cup \psi) &= (\phi)^\otimes + (\psi)^\otimes - (\phi)^\otimes \otimes (\psi)^\otimes \\ (C\phi) &= 1 - (\phi)^\otimes \\ a_i, a_j &\in \Omega \\ (a_i \cap a_j)^\otimes &= \begin{cases} a_i \otimes a_j, & a_i \neq a_j \\ a_i, & a_i = a_j \end{cases} \\ (a_i \cup a_j)^\otimes &= a_i + a_j - (a_i \cap a_j)^\otimes \\ (Ca_i) &= 1 - a_i \end{aligned} \quad (3)$$

The result of any operations on IBA elements is IBA element. GBP maps elements into unit interval. These two properties enable IBA's applications to problems that can be solved by fuzzy logic. According to Radojevic⁴² the GBP is a polynomial whose variables are elements of Boolean algebra as well as the operators' standard +, standard – and generalized product \otimes . Generalized product is defined as any function that maps $[0,1] \times [0,1]$ on the unit interval and is a subclass to the t-norm with add-on non-negativity axiom.⁴³ It is important that the function implementation for generalized product depends on the nature of elementary. More precisely, if elements are of the same nature, generalized product is preferable to be \min ($a_i \otimes a_j = \min(a_i, a_j) = a_i$), while if they are not of the same nature, product operator should be used. In accordance with structural transformation given in Eq. (3), excluded middle axiom is satisfied (Eq. 4), as well as contradiction axiom (Eq. 5):

$$\begin{aligned}\mu_A(x) \vee \overline{\mu_A(x)} &= \mu_A(x) + (1 - \mu_A(x)) - \mu_A(x) \cdot (1 - \mu_A(x)) \\ &= \mu_A(x) + 1 - \mu_A(x) - \mu_A(x) + \mu_A(x) \otimes \mu_A(x) = \\ &= \mu_A(x) + 1 - \mu_A(x) - \mu_A(x) + \mu_A(x) = 1\end{aligned}\quad (4)$$

$$\begin{aligned}\mu_A(x) \wedge \overline{\mu_A(x)} &= \mu_A(x) \cdot (1 - \mu_A(x)) \\ &= \mu_A(x) - \mu_A(x) \otimes \mu_A(x) = 0\end{aligned}\quad (5)$$

In the Boolean consistent approach, values are assigned only after the final structure is established. Since all Boolean axioms hold on a structural level, they will hold on the value level as well. In certain cases, this crucial difference between Boolean consistent fuzzy logic and conventional one leads to different results. The difference in results is most obvious when the logical expression includes negation.

4. Fuzzy Inference IBA Systems for Credit Scoring Classification

In this research we have implemented a fuzzy inference system based on Boolean consistent fuzzy logic, and this is a first such an application for credit scoring. We use Boolean consistent fuzzy logic because its application was justified in numerous papers in different fields^{44, 45, 46, 47, 48}.

A novelty in this research is that Boolean consistent fuzzy inference system is used for evaluation of credit scores. Furthermore, previous implementation of Boolean consistent fuzzy inference system used for prediction of disease is based on one fuzzy rule. In most cases, fuzzy system consists of multiple rules. We implement FIS for credit scoring and we base reasoning on two rules. Moreover, we aggregate rules according to their interdependencies. If rules are mutually dependent, aggregation should be implemented as weighted sum. Otherwise, aggregation can be implemented as max.

In order to create fuzzy inference system, it is necessary to create decision rules (i.e. fuzzy rules) and to define the shape and parameters of membership functions. In this initial step domain expert molds his experience and expertise into decision making system. Fuzzy logic enables transformation of those decision rules into logical expression.

After domain experts create decision rules and define the shape and parameters of membership functions, transformation of rules can be performed using conventional and Boolean consistent fuzzy logic.

4.1. Problem and variables description

The aim of this paper is to define fuzzy inference system for credit scoring of MSME. We define FIS based on consistent fuzzy logic, as well as FIS based on conventional fuzzy logic, in order to compare the accuracy of classification of the proposed system.

We use performance measures data from CUBE database, to determine credit scores based on these two approaches and we compare their properties. CUBE database is a private dataset of Serbian small and medium enterprises performance measures and credit scores. We had records entries for 100 firms, for the year 2011.

In order to define FIS for credit scoring, we base our calculation on credit analysis. Credit analysis differs dependent on the maturity of the loan, which is mainly based on the ratio analysis. In order for us to create FIS which is going to make banks more comfortable to lend to MSME, we are going to use a combination of different group of ratios, which assess loan applications for different terms and gives a complete picture of the enterprise's health and repayment ability. Credit scoring of MSME represents a challenging task, since there is a limited set of available information. Right combination of ratios should allow us to surpass this limitation.

If an enterprise seeks short-term loan, credit analysis may be focused on working capital cycle, liquidity position of an enterprise and quality of its assets*. Also, credit analysis should focus on the availability of the cash flow. Credit assessment for a medium-term loan should analyze borrowers' operations and determine whether earnings would be stable even after the term of the loan. Credit analysis of the long-term loans examines historical and projected cash flows, and whether the debt repayment schedule is matching those projected cash flows. Crucial is that an enterprise has long-term earnings potential[†].

Credit analysis tackles the areas of profitability, performance, liquidity, and solvency. Profitability ratios are important for credit analysts because they can give an insight into whether the enterprise would be able to respond to adverse business conditions, generate enough profits in order to service debt.⁴ Performance ratios should give a picture about management's ability to generate earnings with enterprises capital, sales or total assets.⁴ Solvency ratios are important for credit analysis, because they give an insight into whether the enterprise is extending itself with high levels of debt.⁴ Besides solvency, credit analysts always check for liquidity of an enterprise since both areas are critical for its profitability. Insufficient cash generation can lead an enterprise into bankruptcy due to poor long-term planning.⁴ Lenders are also interested in whether credit is extended to companies customers, and by the use of efficiency ratios they check how working capital resources are managed. If an enterprise has a high business risk, and its earnings are prone to decline, lenders want to see the low level of debt.⁴

On the bases of previous discussion, expert knowledge, and research in that field, the credit score (CS) is estimated based on following input variables^{49, 4}:

- EBITDA (EBITDA margin) is a profitability ratio, which can be used as a proxy for cash flow to sales. It is useful to check the trend of this ratio, since it is favorable for this ratio to remain stable or increase in time,
- OROA (operating return on assets) is another ratio from the group of profitability ratios, which is also

called basic earning power ratio. It represents a measure of the operating income that results from the enterprises investment in total assets,

- QR (quick ratio) is a liquidity ratio, which measures enterprises ability to pay current obligations with its most liquid assets. This ratio shows whether enterprise is capable to generate cash for its immediate needs,
- D/E (Debt-to-equity ratio) is a ratio which tells us about financial strength or solvency of an enterprise. Companies with high levels of debt in their capital structure will have a high value of this ratio,
- NDAR (Number of days of receivables) is an efficiency ratio, which shows the number of days between the date of the sale and the date when the cash is collected,
- NDIR (Number of days of inventory) is another efficiency ratio, which indicates management's ability to manage inventory. Raw materials and finished goods can be liquidated with ease, but work in progress is not easy to liquidate if it becomes compulsory,
- OC (Operating cycle) is efficiency ratio, which measures how long does it takes for the enterprise to receive back cash from its investment in inventory and accounts receivable, when purchases can be done on credit,
- ROA (Return on assets) is a profitability ratio which represents a measure of what an enterprise receives, based on the investments in assets that she made.

4.2. Rules

Fuzzy inference system uses fuzzy rules that enable intuition and experience to be imputed into the model. Proposed FIS is based on two fuzzy rules established by financial professionals. Both rules combine profitability, liquidity, efficiency, and solvency ratios. Also, based on their previous expertise, financial professionals defined the shape and parameters of membership functions. The output of each rule is aggregated into the single value, which presents credit score.

The EBITDA margin could be seen as a measure of size/profitability.^{5,4} First rule suggests that if either size is not high or if there is a problem with liquidity, then if operating return on assets is high it means that the enterprise is capable to generate earnings. Also, if the number of days in inventory is low, it doesn't take long

* Those characteristics could be checked if an analyst look at for example: number of days of receivables', number of days of inventory, and quick ratio, among others

† These areas can be analyzed by using EBITDA margin, operating return on assets, return on assets, and debt to equity ratio, among others.

for enterprise to convert inventory to sales, and if at the same time debt is not high, credit score will be high since entity will be able to repay its debt obligations. In other words, if EBITDA margin and quick ratio are high, then credit score is high, or if EBITDA margin and quick ratio are not high, then in order for credit score to be high, operating return on assets needs to be high, the number of days in inventory, debt-to-equity ratio needs to be low. If an enterprise is liquid and profitable then its credit score is high because it will be able to repay its obligations of various terms. Mathematically, first rule could be written as follow:

Rule 1

If (EBITDA is high AND QR is high)
OR
((NOT (EBITDA is high AND QR is high))
AND
(OROA is high AND NDIR is low AND D/E is low))
Then CS_1 is high

Second rule suggests that a high credit score is based on the two elements. If an enterprise is generating high return on existing assets and if operating cycle is not a long one, or if operating cycle is long but if the duration is not influenced by efficiency of inventory or receivables management and if at the same time debt to equity is not high, credit score will be high. If

profitability and efficiency are high and at the same time if there are not high levels of debt in the capital structure, then the probability of default will not be high and therefore credit score will be high. Or if operating cycle is on a long side, but there is efficiency in managing inventory and receivables, and if the debt is not high, credit score is high due to efficiency and low level of debt.

Rule 2

If ((ROA is high AND OC is low)
OR
((NOT (OC is low)) AND (NDIR is low AND NDAR is low)))
AND D/E is low
Then CS_2 is high

For the aggregation (CS) of the outputs of these two rules (CS_1 and CS_2), linear combination and max method can be used. Since the rules are related to the different aspects of credit scoring elements, the use of linear combination is preferable, i.e. both of aspects should be included in final credit score estimation to some extent.

Financial experts assigned weights 0.55 and 0.45 for credit score 1 (CS_1) and credit score 2 (CS_2), respectively. The specific fuzzy inference system for credit scoring proposed in this paper is shown in Figure1.

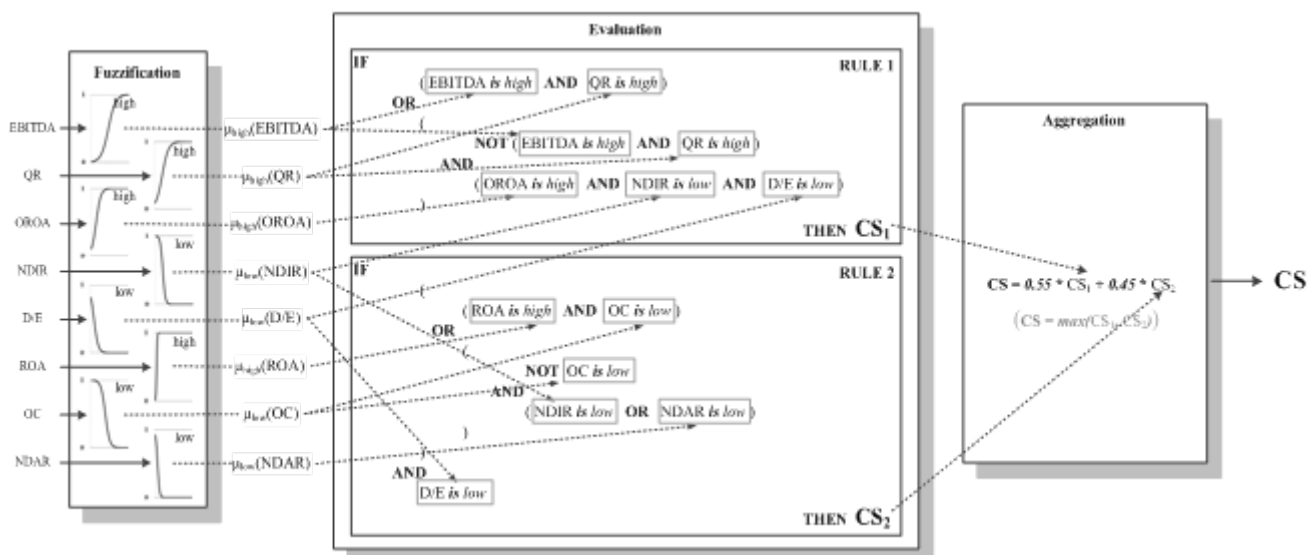


Fig. 1. Fuzzy inference system for credit scoring

4.3. Transformation of the rules

Conventional and Boolean consistent fuzzy logic can be applied for rules evaluation. The transformation of a Boolean function into corresponding GBP requires estimation of a structure (according to Eq. 3). Radojevic⁴¹ and Radojevic⁴² present detailed description how transformations must be executed.

Rules defined by financial professional, could be written as logical expressions Eq. (6), Eq. (7):

Rule 1:

$$\begin{aligned} & (EBITDA \wedge QR) \vee \\ & (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D/E)) \end{aligned} \quad (6)$$

Rule 2:

$$\begin{aligned} & (((OROA \wedge OC) \vee (\perp OC \wedge (NDIR \vee NDAR))) \wedge D/E \end{aligned} \quad (7)$$

After set of transformations is performed, the first rule for credit score classification, for the Boolean consistent approach is given in Eq. (8):

$$\begin{aligned} & ((EBITDA \wedge QR) \vee (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D/E)))^{\otimes} \\ & = EBITDA \otimes QR + OROA \otimes NDIR \otimes D/E \\ & - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D/E \end{aligned} \quad (8)$$

The final structure for conventional approach is given in Eq. (9):

$$\begin{aligned} & ((EBITDA \wedge QR) \vee (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D/E)))^{\otimes} \\ & = EBITDA \otimes QR + OROA \otimes NDIR \otimes D/E \\ & - 2 \cdot EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D/E \\ & + EBITDA \otimes QR \otimes EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D/E \end{aligned} \quad (9)$$

Similarly, for the second rule, transformations give the following result Eq. (10), for Boolean consistent approach:

$$\begin{aligned} & (((OROA \wedge OC) \vee (\perp OC \wedge (NDIR \vee NDAR))) \wedge D/E)^{\otimes} \\ & = OROA \otimes OC \otimes D/E + NDIR \otimes D/E + NDAR \otimes D/E \\ & - NDIR \otimes NDAR \otimes D/E - OC \otimes NDIR \otimes D/E \\ & - OC \otimes NDAR \otimes D/E - OC \otimes NDIR \otimes NDAR \otimes D/E \end{aligned} \quad (10)$$

And for conventional approach Eq. (11):

$$\begin{aligned} & (((OROA \wedge OC) \vee (\perp OC \wedge (NDIR \vee NDAR))) \wedge D/E)^{\otimes} \\ & = OROA \otimes OC \otimes D/E + NDIR \otimes D/E + NDAR \otimes D/E \\ & - NDIR \otimes NDAR \otimes D/E - OC \otimes NDIR \otimes D/E \\ & - OC \otimes NDAR \otimes D/E + OC \otimes NDIR \otimes NDAR \otimes D/E \\ & - OROA \otimes OC \otimes NDIR \otimes D/E - OROA \otimes OC \otimes NDAR \otimes D/E \\ & + OROA \otimes OC \otimes NDIR \otimes NDAR \otimes D/E + \\ & OROA \otimes OC \otimes OC \otimes NDIR \otimes D/E \\ & + OROA \otimes OC \otimes OC \otimes NDAR \otimes D/E \\ & - OROA \otimes OC \otimes OC \otimes NDIR \otimes NDAR \otimes D/E \end{aligned} \quad (11)$$

These transformations need not to be performed manually; instead the software could be used.⁵⁰ After transformation has been done, the second step is value level. On this level, membership values are being assigned, and generalized product is implemented as standard product. Each rule gives its own output value, which is being aggregated into a single value using linear combination.

Aggregated output value can be categorized in one of the three predefined classes. First class (Approved ≥ 75) refers to those enterprises that are credit viable. Second class ($75 > \text{Gray area} \geq 49$) is reserved for enterprises for which we need additional information in order to be able to make an appropriate credit decision. Enterprises assigned to the third class (Default < 49) are those that do not meet eligibility criteria for loan approval. Same cut-off points were used for categorization into three classes, for both, proposed and conventional fuzzy inference system. This principal is used, in order for us to be able to determine which system produces more reliable results.

4.4 Analysis of results

This application of FIS is significant, since very few researches deals with credit scoring of MSME. Literature review showed that previous researches are mostly connected to the scoring of individual credit card holders or to determination of credit rating of listed firms^{8, 15, 19}.

Financial performance measures included in the database are: earnings before interest, taxes, depreciation, and amortization margin (EBITDA), operating return on assets, net profit margin, return on equity, return on assets, quick ratio, cash ratio, debt-to-equity ratio, fixed assets to equity capital ratio, fixed assets to long-term liabilities ratio, fixed assets to total assets ratio, number of days in accounts receivable ratio, number of days in inventory ratio, number of days in

accounts payable, operating cycle, fixed assets turnover, total assets turnover, equity turnover. Entries for all companies were not complete. After eliminating 19 missing cases, our sample consists of 81 companies. Credit score data represent information that is internal to the bank, and it is difficult to obtain such a data. Therefore, we are using a database that provided some amount of records. We acknowledge that our dataset is not a large one, but as it is already stated, our motivation is to fill the gap in existing literature regarding loan approval to MSME, and information about this sector is scarce, and that stressed the importance of this research even more. Nonetheless, our proposed system is tested and shows good results on a small sample, but it can also be implemented for any sample size.

We apply Boolean consistent fuzzy inference system in order to determine credit score of MSME, and we compare its performance to the conventional fuzzy inference system. The experimental results show that proposed model outperforms conventional one. Out of 80 enterprises in our sample, 59 were classified correctly by proposed model, so its accuracy is 73.75%, compared to 52 classified by the conventional model, which accuracy was 65%.

Since fuzzy inference systems were used for classification, confusion matrix is created in order to compare the results obtained by conventional and Boolean consistent fuzzy logic. Matrices are displayed in Table 1.

Table 1: Confusion matrices

Conventional Fuzzy logic	Gray area	Default
Gray area	3 (3.75%)	26 (32.50%)
Default	2 (2.50%)	49 (61.25%)
Boolean consistent Fuzzy logic	Gray area	Default
Gray area	9 (11.25%)	19 (23.75%)
Default	2 (2.50%)	50 (62.50%)

One result is omitted from the confusion matrices, since only 1 enterprise was classified in the first group of credit viable ones. Such results are not unexpected, since our database consists of MSME in frontier market, and as such their profitability and development is not on a high level. Hence, loan approval to such entities cannot be done with certainty. We focus our analysis to

the enterprises that were classified into second (gray area) and third group (default). Therefore, our calculations of accuracy, precision, and recall are based on the 80 enterprises.

If we look at the Boolean consistent fuzzy logic results in a confusion matrix, we can see that 73.75% of enterprises were classified correctly and 26.25% was not classified properly. On the other hand, conventional fuzzy logic correctly classified 65% of enterprises and 35% did not correctly classify.

Precision of conventional FIS is 10.34%, while precision of consistent FIS is 32.14%. Recall for conventional FIS is 60% and of consistent one is 81.82%. We can see that both precision and recall display much better results for consistent FIS. These two performance measures have important implications for banks. Precision describes number of enterprises in gray area that are predicted and actually classified there, compared to the total number of gray area enterprises that are predicted to belong there. Precision suggests that consistent FIS recognizes better which enterprises belong to the gray area. Recall describe actual and predicted number of enterprises in the gray area compared to the actual number of gray area enterprises. Recall results suggest that, if a bank uses proposed credit scoring model, it will be able to rightfully approve more loans to the MSME, which in turn will increase banks' profits.

For example, for an enterprise from our sample with following values of membership functions:

$\mu_{EBITDA}=0.0502$, $\mu_{ROA}=0.5000$, $\mu_{ROA}=0.9561$, $\mu_{QR}=0.9820$, $\mu_{D/E}=0.7408$, $\mu_{NDAR}=0.2769$, $\mu_{NDIR}=0.9890$, $\mu_{OC}=0.8022$, classical approach will give the value of 48.01 (belongs to the default area) while consistent FIS gives 53.97 (belongs to the gray area). If we look at the values of membership functions alone, we can see that there is a potential for a credit approval and that bank would need additional information. That is also more in accordance with the results obtained with Boolean consistent FIS.

The biggest difference between conventional and consistent FIS can be seen if we look at the Gray area. Conventional fuzzy dismissed more loan applications than consistent fuzzy, where they should be classified in the gray area. Consistent fuzzy reasoning shows better results in this respect. This finding is important since we are trying to improve financing for MSME especially in undeveloped countries.

5. Conclusion

In this research we propose Boolean consistent fuzzy inference system for credit scoring of MSME. To the best of our knowledge, this is the first time that Boolean consistent FIS is proposed for credit scoring purposes. We use Boolean consistent fuzzy logic, since this approach has proved to be valid in several other areas.

Fuzzy logic has been widely used in the decision making process in various domains. Fuzzy logic is used to express expert knowledge in the form of fuzzy rules. Since no conventional fuzzy set theory is in the Boolean frame (excluded middle and contradiction are not satisfied on the whole interval $[0, 1]$), it is proposed for rules in our model, to be evaluated using Boolean consistent fuzzy logic.

Boolean consistent fuzzy logic differ two levels. Consistent approach requires set of structural transformations to be done before value level, which is opposite to the conventional approach. Hence, Boolean axioms will hold and partial order at the value level is persevered. In other words, Boolean consistent fuzzy logic guarantees that all Boolean axioms are valid on the whole interval $[0, 1]$ not only on the set $\{0, 1\}$, and that leads to different results.

Empirical results show that this property generates better results compared to the conventional approach. Accuracy of our model is 73.75%, compared to conventional FIS, which accuracy was 65%.

When we look at the application of standard FIS, commonly fuzzy reasoning is based on more than one rule. According to our knowledge, consistent FIS developed in previous researches base reasoning on one fuzzy rule. We extend this and in contrast to prior work, we implement multiple decision rules.

Also, in contrast to most researches we use more ratios of different categories in order to more accurately classify enterprises in accordance to their creditworthiness. That enables our credit scoring system to assess MSME application for loans of different maturities.

Further researches should test this model on different data sets. We acknowledge that our dataset had a limited amount of data available regarding enterprises and ratios. Although we have incorporated different category of ratios into our model for capturing overall performance (which were available to us) perhaps use of different ratios could have more predictive power of the

creditworthiness' of an entity. Also, if our dataset would contain more observations, it could be reasonable to fine tune input membership functions based on input-output dataset. Further researches can be moved towards implementation of neuro-fuzzy system based on consistent fuzzy logic for credit scoring purposes.

References

1. K. C. Chakrabarty, Credit scoring – an effective way to ensure availability of timely and adequate credit to Micro and Small Enterprises (MSEs). Resource document. Keynote address at the Training Workshop on credit scoring model with support from IFC for MSE lending, Mumbai, 29 November, 2013. <http://www.bis.org/review/r131205b.pdf>. Accessed 01 July 2016.
2. D. P. Joshi, Credit scoring. Resource document. Speech at the launch of the credit scoring model at Mumbai, 8th of May, 2014. <http://www.bis.org/review/r140514d.pdf>. Accessed 01 July 2016.
3. Basel, Principles for the management of credit risk: Consultative paper. Resource document. Basel Committee on Banking Supervision (1999) <http://www.bis.org/publ/bcbs54.pdf>. Accessed 01 July 2016.
4. J. Colquitt, *Credit risk management: How to avoid lending disasters and maximize earnings*, 3rd edn. (The McGraw Hill Companies, Inc., USA, 2007).
5. J. Bessis, *Risk Management in Banking*, 2nd edn. (John Wiley & Sons Ltd., Chichester, 2002).
6. M. Ala'rajand and M. F. Abbod, A new hybrid ensemble credit scoring model based on classifiers consensus system approach, *Expert Syst Appl* **64** (2016) 36-55. <http://dx.doi.org/10.1016/j.eswa.2016.07.017>
7. V. S. Desai, J. N. Crookand G. A. Overstreet, A comparison of neural networks and linear scoring models in the credit union environment, *Eur J Oper Res* **95**(1) (1996) 24-37.
8. T. Harris, Credit scoring using the clustered support vector machine, *Expert Syst Appl* **42**(2) (2015) 741-750. <http://dx.doi.org/10.1016/j.eswa.2014.08.029>
9. H. Ince and B. Aktan, A comparison of data mining techniques for credit scoring in banking: A managerial perspective, *J Bus Econ Manag* **10**(3) (2009) 233-240. <http://dx.doi.org/10.3846/1611-1699.2009.10.233-240>
10. T. Jacobson and K. Roszbach, Bank lending policy, credit scoring and value-at-risk, *J bank financ* **27**(4) (2003) 615-633.
11. L. J. Kao, C. C. Chiu and F. Y. Chiu, A Bayesian latent variable model with classification and regression tree approach for behavior and credit scoring, *Knowl-Based Syst* **36** (2012) 245-252. <http://dx.doi.org/10.1016/j.knosys.2012.07.004>

12. F. N. Koutanaei, H. Sajedi and M. Khanbabaei, A hybrid data mining model of feature selection algorithms and ensemble learning classifiers for credit scoring, *J Retailing Consum Serv* **27** (2015) 11-23. <http://dx.doi.org/10.1016/j.jretconser.2015.07.003>
13. V. Kozeny, Genetic algorithms for credit scoring: Alternative fitness function performance comparison. *Expert Syst Appl* **42**(6) (2015) 2998-3004. <http://dx.doi.org/10.1016/j.eswa.2014.11.028>
14. T. S. Lee, C. C. Chiu, C. J. Lu and I. F. Chen, Credit scoring using the hybrid neural discriminant technique. *Expert Syst Appl* **23**(3) (2002) 245-254.
15. T. S. Lee, C. C. Chiu, Y. C. Chou and C. J. Lu, Mining the customer credit using classification and regression tree and multivariate adaptive regression splines, *Comput Stat Data An* **50**(4) (2006) 1113-1130. doi:10.1016/j.csda.2004.11.006
16. S. Lessmann, B. Baesens, H. V. Seow and L. C. Thomas, Benchmarking state-of-the-art classification algorithms for credit scoring: An update of research, *Eur J Oper Res* **247**(1) (2015) 124-136. <http://dx.doi.org/10.1016/j.ejor.2015.05.030>
17. H. Sabzevari, M. Soleymani and E. Noorbakhsh, A comparison between statistical and data mining methods for credit scoring in case of limited available data, in *Proc. 3rd CRC Credit Scoring Conference* (UK, Edinburgh, 2007).
18. D. West, Neural network credit scoring models, *Comput Oper Res* **27**(11) (2000) 1131-1152.
19. H. Xiao, Z. Xiao and Y. Wang, Ensemble classification based on supervised clustering for credit scoring, *Appl Soft Comput* **43** (2016) 73-86. <http://dx.doi.org/10.1016/j.asoc.2016.02.022>
20. Z. Zhao, S. Xu, B. H. Kang, M. M. J. Kabir, Y. Liu, R. and Wasinger, Investigation and improvement of multi-layer perceptron neural networks for credit scoring. *Expert Syst Appl* **42**(7) (2015) 3508-3516. <http://dx.doi.org/10.1016/j.eswa.2014.12.006>
21. Z. Huang, H. Chen, C. J. Hsu, W. H. Chen and S. Wu, Credit rating analysis with support vector machines and neural networks: a market comparative study, *Decis Support Syst* **37**(4) (2004) 543-558. doi:10.1016/S0167-9236(03)00086-1
22. E. I. Altman, Financial ratios, discriminant analysis, and the prediction of corporate bankruptcy, *J Financ*, **23**(4) (1968) 589-609.
23. J. H. Myers and E. W. Forgy, The development of numerical credit evaluation systems, *J Am Stat assoc* **58**(303) (1963) 799-806.
24. M. J. Gardner and D. L. Mills, Evaluating the likelihood of default on delinquent loans, *Financ Manage* **18**(4) (1989) 55-63.
25. W. E. Henley and D. J. Hand, A k-nearest-neighbor classifier for assessing consumer credit risk, *Statistician* **45**(1) (1996) 77-95.
26. A. De Servigny and O. Renault, *Measuring and Managing Credit Risk* (McGraw Hill, New York, 2004).
27. A. Marcano-Cedeno, A. Marin-De-La-Barcelona, J. Jiménez-Trillo, J. A. Pinuela and D. Andina, Artificial metaplasticity neural network applied to credit scoring, *Int j neural syst* **21**(04) (2011) 311-317. DOI: 10.1142/S0129065711002857
28. R. Alejo, V. Garcia, A. I. Marques, A. J. Sanchez and J. A. Antonio-Velazquez, Making accurate credit risk predictions with cost-sensitive mlp neural networks, in *Management Intelligent Systems* (Springer International Publishing, 2013), pp. 1-8. DOI: 10.1007/978-3-319-00569-0_1
29. A. Khashman, Neural networks for credit risk evaluation: Investigation of different neural models and learning schemes, *Expert Syst Appl* **37**(9) (2010) 6233-6239. <http://dx.doi.org/10.1016/j.knosys.2012.07.004>
30. C. K. Leong, Credit Risk Scoring with Bayesian Network Models, *Comput Econ*, **47** (2016) 423-446. DOI 10.1007/s10614-015-9505-8
31. H. Fang, Adaptive neurofuzzy inference system in the application of the financial crisis forecast, *Int J Innov Manage Tech* **3**(3) (2012) 250-254.
32. S. Piramuthu, Financial credit-risk evaluation with neural and neurofuzzy systems, *Eur J Oper Res* **112**(2) (1999) 310-321.
33. R. Malhotra and D. K. Malhotra, Differentiating between good credits and bad credits using neuro-fuzzy systems. *Eur J Oper Res* **136**(1) (2002) 190-211.
34. Y. S. Chen and C. H. Cheng, Hybrid models based on rough set classifiers for setting credit rating decision rules in the global banking industry, *Knowl-Based Syst* **39** (2013) 224-239. <http://dx.doi.org/10.1016/j.knosys.2012.11.004>
35. M. Bellalah, S. Zouari and O. Levyne, The performance of hybrid models in the assessment of default risk, *Econ Model* **52** (2016) 259-265. <http://dx.doi.org/10.1016/j.econmod.2014.10.051>
36. A. Ekinici and H. I. Erdal, Forecasting Bank Failure: Base Learners, Ensembles and Hybrid Ensembles, *Comput Econ* **49** (2017) 677-686. DOI 10.1007/s10614-016-9623-y
37. H. Jo, I. Han and H. Lee, Bankruptcy prediction using case-based reasoning, neural networks, and discriminant analysis, *Expert Syst Appl* **13**(2) (1997) 97-108.
38. F. M. Tseng and Y. C. Hu, Comparing four bankruptcy prediction models: Logit, quadratic interval logit, neural and fuzzy neural networks, *Expert Syst Appl* **37**(3) (2010) 1846-1853. doi:10.1016/j.eswa.2009.07.081
39. L. A. Zadeh, Fuzzy sets, *Inform control* **8**(3) (1965) 338-353.
40. L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning – I, *Inform sciences* **8**(3) (1975) 199-249.
41. D. Radojevic, Interpolative realization of Boolean algebra as a consistent frame for gradation and/or fuzziness, in *Forging the new frontier (Studies in Fuzziness and Soft Computing)*, eds. M. Nikraves, J.

- Kacprzyk, L. A. Zadeh (Springer-verlag, 2008b) Vol. 104, pp. 295-317.
42. D. Radojević, Boolean Frame is Adequate for Treatment of Gradation or Fuzziness Equally as for Two-Valued or Classical Case, in *Proc. 4th Serbian-Hungarian Joint Symp on Intelligent Systems–SISY* (2006) pp. 43-57.
 43. D. Radojevic, Logical aggregation based on interpolative Boolean algebra, *Mathware Soft Comput* **15**(1) (2008c) 125-141.
 44. I. Dragovic, N. Turajlic, D. Radojevic, B. Petrovic, Combining Boolean consistent fuzzy logic and AHP illustrated on the web service selection problem, *Int J Comput Int Sys* **7**(supl) (2014) 84-93. doi:10.1080/18756891.2014.853935
 45. K.Mandic, B.Delibasic and D.Radojevic, An Application of the Integrated IBA-TOPSIS Model in Supplier Selection, *Int J Decis Support Syst Tech (IJDSST)* **7**(1)(2015)15-30.
 46. K. Mandic, B. Delibasic and D. Radojevic, Supplier selection using Interpolative Boolean algebra and TOPSIS method, in *Proc. 14th Joint International Conference of the INFORMS GDN Section and the EURO Working Group on DSS* (Toulouse, France, 10-13 June 2014), pp. 134.
 47. A. Poledica, P. Milosevic, I. Dragovic, B. Petrovic and D. Radojevic, Modeling consensus using logic-based similarity measures, *Soft Comput* **19**(11) (2015) 3209-3219. doi:10.1007/s00500-014-1476-5
 48. I. Dragovic, N. Turajlic, D. Pilcevic, B. Petrovic and D. Radojevic, A Boolean consistent fuzzy inference system for diagnosing diseases and its application for determining peritonitis likelihood, *Comput math method m* (2015) article ID 147947. <http://dx.doi.org/10.1155/2015/147947>
 49. M. R. Clayman, M. S. Fridson and G. H. Troughton, *Corporate Finance: A Practical Approach* (John Wiley & Sons, Inc., New Jersey, 2008).
 50. P. Milosevic, B. Petrovic, D. Radojevic and D. Kovacevic, A software tool for uncertainty modeling using Interpolative Boolean algebra, *Knowl-Based Syst* **62** (2014) 1-10. <http://dx.doi.org/10.1016/j.knosys.2014.01.019>.

Appendix A.

Detailed transformation steps of the first rule for credit score classification Eq. (8), for Boolean consistent approach are given in Eq. (A.1):

$$\begin{aligned}
 & ((EBITDA \wedge QR) \vee (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E)))^{\otimes} \\
 &= (EBITDA \wedge QR)^{\otimes} + (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E))^{\otimes} - (EBITDA \wedge QR)^{\otimes} \otimes (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E))^{\otimes} \\
 &= EBITDA \otimes QR + ((1 - EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) - (EBITDA \otimes QR) \otimes ((1 - EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) \\
 &= EBITDA \otimes QR + (OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E) \\
 &\quad - ((EBITDA \otimes QR - EBITDA \otimes QR \otimes EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) \\
 &= EBITDA \otimes QR + (OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E) \\
 &\quad - (EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E) \\
 &= EBITDA \otimes QR + OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E \\
 &\quad - (EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E) \\
 &= EBITDA \otimes QR + OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E
 \end{aligned} \tag{A.1}$$

And for conventional approach Eq. (9), detailed transformation is given in Eq. (A.2):

$$\begin{aligned}
 & ((EBITDA \wedge QR) \vee (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E)))^{\otimes} \\
 &= (EBITDA \wedge QR)^{\otimes} + (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E))^{\otimes} - (EBITDA \wedge QR)^{\otimes} \otimes (\perp (EBITDA \wedge QR) \wedge (OROA \wedge NDIR \wedge D / E))^{\otimes} \\
 &= EBITDA \otimes QR + ((1 - EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) - (EBITDA \otimes QR) \otimes ((1 - EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) \\
 &= EBITDA \otimes QR + (OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E) \\
 &\quad - ((EBITDA \otimes QR - EBITDA \otimes QR \otimes EBITDA \otimes QR) \otimes (OROA \otimes NDIR \otimes D / E)) \\
 &= EBITDA \otimes QR + OROA \otimes NDIR \otimes D / E - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E \\
 &\quad - EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E + EBITDA \otimes QR \otimes EBITDA \otimes QR \otimes OROA \otimes NDIR \otimes D / E
 \end{aligned} \tag{A.2}$$

Similarly for the second rule, detailed transformations of equation Eq. (10), is presented in Eq. (A.3):

$$\begin{aligned}
 & (((OROA \wedge OC) \vee (\perp OC \wedge (NDIR \vee NDAR))) \wedge D / E)^{\otimes} \\
 &= ((OROA \wedge OC)^{\otimes} + (\perp OC \wedge (NDIR \vee NDAR))^{\otimes} - (OROA \wedge OC)^{\otimes} \otimes (\perp OC \wedge (NDIR \vee NDAR))^{\otimes}) \otimes D / E \\
 &= \left(OROA \otimes OC + ((1 - OC) \otimes (NDIR + NDAR - NDIR \otimes NDAR)) \right. \\
 &\quad \left. - (OROA \otimes OC) \otimes ((1 - OC) \otimes (NDIR + NDAR - NDIR \otimes NDA)) \right) \otimes D / E \\
 &= \dots \\
 &= OROA \otimes OC \otimes D / E + NDIR \otimes D / E + NDAR \otimes D / E - NDIR \otimes NDAR \otimes D / E \\
 &\quad - OC \otimes NDIR \otimes D / E - OC \otimes NDAR \otimes D / E - OC \otimes NDIR \otimes NDAR \otimes D / E
 \end{aligned} \tag{A.3}$$

And for conventional approach displayed in Eq. (11), detailed transformations can be seen in Eq. (A.4):

$$\begin{aligned}
 & \left(\left((OROA \wedge OC) \vee (\perp OC \wedge (NDIR \vee NDAR)) \right) \wedge D / E \right)^{\otimes} \\
 & = \dots \\
 & = OROA \otimes OC \otimes D / E + NDIR \otimes D / E + NDAR \otimes D / E - NDIR \otimes NDAR \otimes D / E \\
 & - OC \otimes NDIR \otimes D / E - OC \otimes NDAR \otimes D / E + OC \otimes NDIR \otimes NDAR \otimes D / E \\
 & - OROA \otimes OC \otimes NDIR \otimes D / E - OROA \otimes OC \otimes NDAR \otimes D / E \\
 & + OROA \otimes OC \otimes NDIR \otimes NDAR \otimes D / E + OROA \otimes OC \otimes OC \otimes NDIR \otimes D / E \\
 & + OROA \otimes OC \otimes OC \otimes NDAR \otimes D / E - OROA \otimes OC \otimes OC \otimes NDIR \otimes NDAR \otimes D / E
 \end{aligned}
 \tag{A.4}$$