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Research Article

Analysis of Cryptocurrency Market by Using q-Rung Orthopair Fuzzy Hypersoft Set Algorithm Based on Aggregation Operators

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One of the most important innovations brought by digitization is the cryptocurrency, also called virtual or digital currency, which has been discussed in recent years and in particular is a new platform for investors. Different types of cryptocurrencies such as Bitcoin, Ethereum, Binance Coin, and Tether do not depend on a central authority. Decision making is complicated by categorization and transmission of uncertainty, as well as verification of digital currency. The weighted average and weighted geometric aggregation operators are used in this article to define a multi-attribute decision-making approach. This work investigates the uniqueness of q-rung orthopair fuzzy hypersoft sets (q-ROFHSS), which respond to instabilities, uncertainty, ambiguity, and imprecise information. This research also covers some fundamental topics of q-ROFHSS. The model offered here is the best option for learning about electronic currency. This study validates the complexity of decision-making problems with different attributes and subattributes to obtain an optimal choice. We conclude that Bitcoin has a diverse set of applications and that crypto assets are well positioned to become an important asset class in decision making.

1. Introduction

The concept of cryptocurrency has been widely used in the past. Cryptocurrencies are also known as digital currencies that use encryption for transaction verification. In 2009, Satoshi Nakamoto [1] generated a cryptocurrency to be a peer-to-peer electronic cash transaction. As a result, the first cryptocurrency, Bitcoin, was founded in 2009. Cryptocurrencies are digital currencies that use the cryptographic approach and are based on blockchain technology [2]. With exponential changes in the cryptocurrency market, buying or selling a cryptocurrency is a difficult task in the online market. To cope with this, we need to analyze the cryptocurrency market by decision-making problem. The process of selecting the best options from a dataset is known as decision making. To make the right decision, several researchers have given a number of concepts. Decisions were developed at the beginning of the era on the basis of accurate numerical datasets, but this resulted in insufficient conclusions that were less applicable to real circumstances. Many researchers used different decision-making models in some branches of mathematics, statistics, and artificial intelligence. According to Urquhart [3], trading activity and significant volatility draw people's attention to Bitcoin. However, it noted that no meaningful results for anticipating volatility could be found via online searches. However, David et al. [4]. worked on the feedback cycles between the socio-eooi signals in the bitcoin economy. And the used vector autoregression to identify two positive feedback loops. In 2020, Ramadani and Devianto [5] developed the forecasting model of Bitcoin price with fuzzy time series Markov chain and Chen logical method. Fuzzy time series can model various types of time series data pattern because this method is free from classical assumption. In the field of fuzzy set theory, Jana et al. [6] developed a dynamic decision-making method based on aggregation operators in complex q-rung orthopair fuzzy environment. In 2022, Palanikumar et al. [7] also developed multi-attribute decision-making problems based on aggregation operators by using Pythagorean neutrosophic normal interval-valued fuzzy sets. After the classical notion of set, Zadeh [8] proposed fuzzy set theory. Bhattacharya and Mukherjee [9] worked on fuzzy set and developed fuzzy relations and fuzzy groups. Yager and Filev [10, 11] in 1994 using fuzzy set developed aggregation operators, fuzzy models, and formal structures. In 1999, Demirci [12] introduced fuzzy functions and their fundamental properties. Atanassov [13] proposed intuitionist fuzzy sets with the condition that the total of these two grades should not exceed unity. In some cases, the sum of membership grade and non-membership grade is greater to one, then intuitionistic fuzzy sets did not completely fulfill this condition. (e.g. $.9 + .6 > 1$), hence intuitionistic fuzzy sets fail. Yager [14] proposed Pythagorean fuzzy sets, an extended version of intuitionist fuzzy sets in which the square sum of the MM and non-membership grades is less than or equal to one. As in the previous study, linear inequalities between membership and non-membership grades are explored. If the decision maker increases the power to 2, however, $.9^2 + .8^2 \nleq 1$ is obtained, suggesting that the Pythagorean fuzzy set theory is also erroneous. Ali et al. [15] used complex interval-valued Pythagorean fuzzy set in green supplier chain management. Ashraf et al. [16] introduced interval-valued picture fuzzy Maclaurin symmetric mean operator as application in decision-making problem. In the instance of q-rung orthopair fuzzy set (q-ROFS) [17–19], the conditions on membership function and non-membership functions are changed to $0 \leq u^q + v^q \leq 1\,(q \geq 1)$. Even for very large values of "q," we can treat the membership and non-membership grades independently to some extent. As a result, q-ROF set has more ability in terms of processing ambiguous data than intuitionistic and Pythagorean fuzzy set. These theories, on the other hand, are unable to account for the parametric values of the alternatives. Molodtsov [20] presented the concept of soft set theory for dealing with unpredictability in a parametric way in order to overcome these restrictions. He identified some mathematical representations and proposed a soft set theory for solving problems. By using the concepts of soft set, Cagman et al. [21] introduced fuzzy soft sets and also created fuzzy aggregation operators and applied them to real-world applications. Using the soft set scheme, Maji [22, 23] created a fuzzy soft set theory and a neutrosophic soft set theory. The previous study focused only on data gaps caused by membership and non-membership values. These assumptions, on the other hand, are unable to cope with the overall inconsistency and inaccuracy of the data. Previous theories fail to handle such situations when characteristics of a group of parameters contain additional subattributes. In order to overcome the restriction indicated above, Smarandache [24] introduced the hypersoft set theme by using the soft set concept. The basics of the hypersoft set, such as complement, non-set, hypersoft subset, and aggregation operators, were then presented by Saeed et al. [25]. Several scholars have looked into different operators and features under the hypersoft set and its expansions [26–30]. The theme of the fuzzy intuitionistic soft set was then extended and a new theme of the intuitionistic fuzzy hypersoft set was established, as well as aggregation operators for solving MADM problems by Zulqarnain [31].

Motivation. The modeling of the decision-making problems requires deep importance on the attributes, and we cannot directly consider or neglect any attribute without considering its importance. In order to deal with more attributes, it is a dire need to get the benefit of the theory hypersoft set (HSS). Since hypersoft sets deals with attributes and subattributes, while soft theory deals only with attributes, and fuzzy hypersoft sets deal with attributes and sub attributes in an uncertain way. So, that is why we choose the field of hypersoft set theory by considering the nature of subattributes. The main goal of our research is to develop a novel aggregation operator for a q-rung orthopair fuzzy hypersoft environment. We have also created an algorithm to explain multi-criteria decision-making situations, as well as a numerical example to show how the suggested technique works in the q-ROFHS context. In the digital market, the selection and evaluation of cryptocurrency is a vital procedure. As a result, more studies using MCDM approaches in the selection of cryptocurrencies are needed to accurately capture the uncertainty of the cryptocurrency market data as well as that of the manufacturer of decision preferences. We propose some operational principles based on the decision formula in terms of q-ROFH set. We then create two aggregation operators, the q-ROFHWA and q-ROFHWG operators, using operational principles. Score and accuracy functions are also provided to compare the q-ROFH set. To handle decision-making concerns, the algorithm's rule is proposed with the help of the proposed operators. Finally, a numerical example is provided to show the method's efficiency.

2. Materials and Methods

This section collects some fundamental elements that will contribute to the compilation of the remaining part of the article: soft set, hypersoft set, and q-rung orthopair fuzzy hypersoft set and their example.

Definition 1 (see [20]). Let $\circledS$ be a set of discourse with attributive set E, and $\doublebarwedge \subseteq E$. A pair $(\looparrowleft, \doublebarwedge)$ is called soft set over $\circledS$, where $\looparrowleft$ is a function such that $\looparrowleft: \barwedge \longrightarrow P^{\circledS}$ and $P^{\circledS}$ represents the family of all possible subsets of $\circledS$. A pair $(\looparrowleft, \doublebarwedge)$ can be defined as $(\looparrowleft, \doublebarwedge) = \{\langle e, \looparrowleft(e)\rangle | e \in \doublebarwedge, \looparrowleft(e) \in F^{\circledS}\}$.

Definition 2 (see [24]). Let $\circledS$ be a universal set with n distinct attributive sets $a_1, a_2, a_3, \ldots, a_{\ddot{n}}$, whose attributive value belong to the sets $\barwedge_1, \barwedge_2, \ldots, \barwedge_{\ddot{n}}$, respectively, such that $\barwedge_i \cap \barwedge_j = \phi$, for all $i, j \in \{1, 2, \ldots, n\}$. A pair $(\looparrowleft, \doublebarwedge)$ is called hypersoft set over $\circledS$, where $\looparrowleft$ is a function such that $\looparrowleft: \doublebarwedge \longrightarrow P(U)$ and $\doublebarwedge = \barwedge_1 \times \barwedge_2 \times \ldots, \times \barwedge_{\ddot{n}}$. A pair $(\looparrowleft, \doublebarwedge)$ can be defined as $(\looparrowleft, \doublebarwedge) = \{\langle \overline{\propto}, \looparrowleft(\overline{\boxtimes})\rangle | \overline{\propto} \in \doublebarwedge, \looparrowleft(\overline{\boxtimes}) \in P(U)\}$.

Definition 3 (see [32]). Let $\circledS$ be universal set and $a_1, a_2, a_3, \ldots, a_n$ be n distinct attributes concerning $\circledS$ whose corresponding attributive values are members of the sets $\barwedge_1, \barwedge_2, \ldots, \barwedge_{\ddot{n}}$, respectively, such that $\barwedge_i \cap \barwedge_j = \phi$, where $i = j$ for each $n > 1$ and $i, j \in \{1, 2, \ldots, n\}$. A pair $(\looparrowleft, \overline{\barwedge})$ is called q-ROFHS set, where $\looparrowleft$ is a mapping $\looparrowleft: \overline{\barwedge} \longrightarrow q - ROFS^{(\circledS)}$ and $\barwedge_1 \times \barwedge_2 \times \cdots \times \barwedge_{\ddot{n}} = \overline{\barwedge} = \{k_1, k_2, \ldots, k_{\ddot{n}}\}$ is a family of subparameters. A pair $(\looparrowleft, \overline{\barwedge})$ can be expressed as $(\looparrowleft, \overline{\barwedge}) = \{(\overline{\propto}, \looparrowleft_{\overline{\barwedge}}(\overline{\ltimes})): \overline{\propto} \in \overline{\barwedge}, \looparrowleft_{\overline{\barwedge}}(\overline{\ltimes}) \in q - ROFS^{\circledS} \in 0, 1\}$, where $\looparrowleft_{\overline{\barwedge}}(\overline{\ltimes}) = \left\{\langle \ddot{y}, \frown_{\overline{\barwedge}(\overline{\ltimes})}(\ddot{y}), \ddot{\frown}_{\overline{\barwedge}(\overline{\ltimes})}(\ddot{y})\rangle | \ddot{y} \in \circledS \text{ and } q \geq 1\right\}$. Here $\frown$ and $\ddot{\frown}$ represent membership and non-membership functions with the restriction $0 \leq (\frown_{\overline{\barwedge}(\overline{\ltimes})}(\ddot{y}))^q + (\ddot{\frown}_{\overline{\barwedge}(\overline{\ltimes})}(\ddot{y}))^q \leq 1$ and $q \geq 1$, where $q - ROFHSN$ can be expressed as $(\looparrowleft, \overline{\barwedge}) = (\frown_{\overline{\barwedge}(\overline{\ltimes}_{ij})}, \ddot{\frown}_{\overline{\barwedge}(\overline{\ltimes}_{ij})})$.

Example 1. Let $\circledS = \{y_1, y_2, y_3, y_4\}$ be the set of four houses under consideration say $\circledS$ and also consider the set of attributes as

(i) $\urcorner_1$ represents location of the house.
(ii) $\urcorner_2$ represents the price of the house.
(iii) $\urcorner_3$ represents number of bedrooms in the house.

Also, $\urcorner_1 = \{\natural_{11} =$ the proximity of important services, $\natural_{12} =$ resale value in future, $\natural_{13} =$ lifestyle$\}$, $\urcorner_2 = \{\natural_{21} = 5{,}000{,}000, \natural_{22} = 8{,}000{,}000\}$, $\urcorner_3 = \{\natural_{31} = 5, \natural_{32} = 4\}$ are sets of corresponding parameters. Suppose $\barwedge_1 = \{\natural_{11}, \natural_{12}\}, \barwedge_2 = \{\natural_{21}\}, \barwedge_3 = \{\natural_{31}, \natural_{32}\}$, and $B_1 = \{\natural_{11}\}, B_2 = \{\natural_{21}, \natural_{22}\}, B_3 = \{\natural_{31}, \natural_{32}\}$ are subsets of $\urcorner_i$ for $i = 1, 2, 3$. Then, $\overline{\barwedge} = \urcorner_1 \times \urcorner_2 \times \urcorner_3$ will contain elements with three tuples, and we will assume $q = 5$:

$$\begin{aligned}
&\overline{\barwedge} = \{\overline{\ltimes}_1, \overline{\ltimes}_2, \overline{\ltimes}_3, \overline{\ltimes}_4, \overline{\ltimes}_5, \overline{\ltimes}_6, \overline{\ltimes}_7, \overline{\ltimes}_8, \overline{\ltimes}_9, \overline{\ltimes}_{10}, \overline{\ltimes}_{11}, \overline{\ltimes}_{12}\},\\
&\overline{\ltimes}_1 = (\natural_{11}, \natural_{21}, \natural_{31}), \overline{\ltimes}_2 = (\natural_{11}, \natural_{21}, \natural_{32}), \overline{\ltimes}_3 = (\natural_{11}, \natural_{22}, \natural_{31}), \overline{\ltimes}_4 = (\natural_{11}, \natural_{22}, \natural_{32}),\\
&\overline{\ltimes}_5 = (\natural_{12}, \natural_{21}, \natural_{31}), \overline{\ltimes}_6 = (\natural_{12}, \natural_{21}, \natural_{32}), \overline{\ltimes}_7 = (\natural_{12}, \natural_{22}, \natural_{31}), \overline{\ltimes}_8 = (\natural_{12}, \natural_{22}, \natural_{32}),\\
&\overline{\ltimes}_9 = (\natural_{13}, \natural_{21}, \natural_{31}), \overline{\ltimes}_{10} = (\natural_{13}, \natural_{21}, \natural_{32}), \overline{\ltimes}_{11} = (\natural_{13}, \natural_{22}, \natural_{31}), \overline{\ltimes}_{12} = (\natural_{13}, \natural_{22}, \natural_{32}).
\end{aligned} \tag{1}$$

Suppose

$$\begin{aligned}
\overline{\ltimes}_1 &= (\natural_{11}, \natural_{21}, \natural_{31}),\\
\overline{\ltimes}_2 &= (\natural_{11}, \natural_{21}, \natural_{32}),\\
\overline{\ltimes}_3 &= (\natural_{12}, \natural_{21}, \natural_{31}),\\
\overline{\ltimes}_4 &= (\natural_{12}, \natural_{21}, \natural_{32}),\\
\overline{b}_1 &= (\natural_{11}, \natural_{21}, \natural_{31}),\\
\overline{b}_2 &= (\natural_{11}, \natural_{21}, \natural_{32}),\\
\overline{b}_3 &= (\natural_{11}, \natural_{22}, \natural_{31}),\\
\overline{b}_4 &= (\natural_{11}, \natural_{22}, \natural_{32}).
\end{aligned} \tag{2}$$

Then, $(\looparrowleft, \overline{\barwedge})$ and $(\looparrowright, \overline{\beta} * * *)$, two q-ROFHS sets, may be expressed as

$$\begin{aligned}
(\looparrowleft, \overline{\barwedge}) &= \left\{\begin{array}{l}
c\langle \overline{\ltimes}_1, \{(y_1, (.6, .8)), (y_2, (.7, .9)), (y_3, (.8, .6)), (y_4, (.5, .9))\}\rangle\\
\langle \overline{\ltimes}_2, \{(y_1, (.5, .8)), (y_2, (.7, .6)), (y_3, (.9, .5)), (y_4, (.9, .8))\}\rangle\\
\langle \overline{\ltimes}_3, \{(y_1, (.6, .8)), (y_2, (.7, .6)), (y_3, (.7, .9)), (y_4, (.6, .9))\}\rangle\\
\langle \overline{\ltimes}_4, \{(y_1, (.9, .7)), (y_2, (.9, .7)), (y_3, (.7, .8)), (y_4, (.8, .9))\}\rangle
\end{array}\right\},\\
(\looparrowright, \overline{\beta}) &= \left\{\begin{array}{l}
c\langle \overline{b}_1, \{(y_1, (.9, .7)), (y_2, (.8, .9)), (y_3, (.7, .9)), (y_4, (.8, .7))\}\rangle\\
\langle \overline{b}_2, \{(y_1, (.7, .8)), (y_2, (.7, .8)), (y_3, (.9, .8)), (y_4, (.9, .7))\}\rangle\\
\langle \overline{b}_3, \{(y_1, (.9, .8)), (y_2, (.8, .9)), (y_3, (.7, .8)), (y_4, (.8, .9))\}\rangle\\
\langle \overline{b}_4, \{(y_1, (.7, .9)), (y_2, (.8, .9)), (y_3, (.9, .8)), (y_4, (.9, .7))\}\rangle
\end{array}\right\}.
\end{aligned} \tag{3}$$

Tables 1 and 2 show the tabular forms of q-ROFHS values.

Definition 4. For two q-ROFHS sets $(\hookleftarrow, \overline{\barwedge})$ and $(\looparrowright, \overline{\beta})$ by a universe of discourse ⓢ, we define $(\hookleftarrow, \overline{\barwedge})$ as a q-ROFHS subset of $(\looparrowright, \overline{\beta})$, defined as $(\hookleftarrow, \overline{\barwedge}) \widetilde{\subseteq} (\looparrowright, \overline{\beta})$, if the following hold.

(1) $\overline{\barwedge} \subseteq \overline{\beta}$.
(2) For any $\overline{\propto} \in \overline{\barwedge}, \hookleftarrow(\overline{\ltimes}) \subseteq \looparrowright(\overline{b})$.

Example 2. In Example 1, we consider these parameters and assume that $(\hookleftarrow, \overline{\barwedge})$ and $(\looparrowright, \overline{\beta})$ are two q-ROFHS sets on $Y = \{y_1, y_2, y_3, y_4\}$. Tabular forms of $(\hookleftarrow, \overline{\barwedge})$ and $(\looparrowright, \overline{\beta})$ are provided in Tables 3 and 4.

It is clear that $(\hookleftarrow, \overline{\barwedge}) \subseteq (\looparrowright, \overline{\beta})$.

Table 1: q-ROFHS values.

$(\hookleftarrow, \overline{\barwedge})$	y_1	y_2	y_3	y_4
$(\natural_{11}, \natural_{21}, \natural_{31})$	(.6, .8)	(.7, .9)	(.8, .6)	(.5, .9)
$(\natural_{11}, \natural_{21}, \natural_{32})$	(.5, .8)	(.7, .6)	(.9, .5)	(.9, .8)
$(\natural_{12}, \natural_{21}, \natural_{31})$	(.6, .8)	(.7, .6)	(.7, .9)	(.6, .9)
$(\natural_{12}, \natural_{21}, \natural_{32})$	(.9, .7)	(.9, .7)	(.7, .8)	(.8, .9)

Table 2: q-ROFHS values.

$(\looparrowright, \overline{\beta})$	y_1	y_2	y_3	y_4
$(\natural_{11}, \natural_{21}, \natural_{31})$	(.9, .7)	(.8, .9)	(.7, .9)	(.8, .7)
$(\natural_{11}, \natural_{21}, \natural_{32})$	(.7, .8)	(.7, .8)	(.9, .8)	(.9, .7)
$(\natural_{12}, \natural_{21}, \natural_{31})$	(.9, .8)	(.8, .9)	(.7, .8)	(.8, .9)
$(\natural_{12}, \natural_{21}, \natural_{32})$	(.7, .9)	(.8, .9)	(.9, .8)	(.9, .7)

3. Aggregation Operators

The score and accuracy function for q-ROFHSNs are discussed in this part, as well as q-ROFHS weighted average and q-ROFHS weighted geometric operators. Furthermore, we discuss the fundamental properties of q-ROFHS weighted averaging and q-ROFHS weighted geometric aggregation operators by utilizing developed q-ROFHSNs.

Definition 5. The score function of q-ROFHSN is defined as

$$\left(\S_{\overline{\ltimes}_{ij}}\right) = \frown_{\overline{\barwedge}}\left(\overline{\ltimes}_{ij}\right) - \ddot{\frown}_{\overline{\barwedge}}\left(\overline{\ltimes}_{ij}\right). \tag{4}$$

Definition 6. The accuracy function of q-ROFHSN is defined as

$$\beta\left(\S_{\overline{\ltimes}_{ij}}\right) = \frown_{\overline{\barwedge}}\left(\overline{\ltimes}_{ij}\right) + \ddot{\frown}_{\overline{\barwedge}}\left(\overline{\ltimes}_{ij}\right). \tag{5}$$

For the comparison purpose of q-ROFHSNs, the following laws are classified:

(1) $S(\S_{\overline{\ltimes}_{ij}}) > S(\widehat{\S}_{\overline{\ltimes}_{ij}})$; then, $\S_{\overline{\ltimes}_{ij}} > \widehat{\S}_{\overline{\ltimes}_{ij}}$.
(2) $S(\S_{\overline{\ltimes}_{ij}}) = S(\widehat{\S}_{\overline{\ltimes}_{ij}})$; then,
(i) If $\beta(\S_{\overline{\ltimes}_{ij}}) > \beta(\widehat{\S}_{\overline{\ltimes}_{ij}})$, then $\S_{\overline{\ltimes}_{ij}} > \widehat{\S}_{\overline{\ltimes}_{ij}}$.
(ii) If $\beta(\S_{\overline{\ltimes}_{ij}}) = \beta(\widehat{\S}_{\overline{\ltimes}_{ij}})$, then $\S_{\overline{\ltimes}_{ij}} = \widehat{\S}_{\overline{\ltimes}_{ij}}$.

Definition 7. Let $\S_{\overline{\ltimes}_k} = (\frown_{\overline{\barwedge}(\overline{\ltimes}_k)}, \ddot{\frown}_{\overline{\barwedge}(\overline{\ltimes}_k)})$ be a q-ROFHNs and $w_i = \{w_1, w_2, \ldots, w_{\ddot{n}}\}$ and $v_i = \{v_1, v_2, \ldots, v_{\ddot{m}}\}$ be the expert weight vectors and selected subattributes, respectively, with the condition that $w_i > 0, \sum_{i=1}^{\ddot{n}} w_i = 1, v_i > 0, \sum_{i=1}^{\ddot{m}} v_i = 1$. The mapping for the q-ROFHWA operator is thus defined as $q-\mathrm{ROFHWA}\colon \Delta^{\ddot{n}} \longrightarrow \Delta$, where Δ is the collection of all q-ROFHNs, provided as

$$q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) = \oplus_{j=1}^{\ddot{m}} v_i \left(\oplus_{i=1}^{\ddot{n}} w_i \S_{\overline{\ltimes}_{ij}}\right). \tag{6}$$

Example 3. Let ⓢ be the set of decision makers to decide best laptop given as ⓢ $= \{y_1, y_2, y_3\}$ and also consider the set of attributes as $\daleth_1$ and $\daleth_2$, where $\daleth_1$ represents laptop type and $\daleth_2$ represents laptop RAM. Then, their corresponding attributive sets can be $\daleth_1 = \{a_{11} = \mathrm{HP}, a_{12} = \mathrm{Dell}\}$, $\daleth_2 = \{a_{21} = 8\,\mathrm{GB}, a_{22} = 16\,\mathrm{GB}, a_{23} = 16\,\mathrm{GB}\}$

Suppose $\barwedge_1 = \{\natural_{11}, \natural_{12}\}, \barwedge_2 = \{\natural_{21}, \natural_{22}\}$. Then, $\overline{\barwedge} = \daleth_1 \times \daleth_2$ will have 2 tuple elements and we suppose $q = 8$.

Then, q-ROFHS set $(\hookleftarrow, \overline{\barwedge})$ can be written as

$$\begin{aligned}
\langle \S_{\overline{\ltimes}_{11}} \rangle &= \langle 0.81, 0.88 \rangle, \\
\langle \S_{\overline{\ltimes}_{12}} \rangle &= \langle 0.95, 0.97 \rangle, \\
\langle \S_{\overline{\ltimes}_{21}} \rangle &= \langle 0.87, 0.84 \rangle, \\
\langle \S_{\overline{\ltimes}_{22}} \rangle &= \langle 0.95, 0.91 \rangle.
\end{aligned} \tag{7}$$

Let $w_i = (0.2, 0.1, 0.4, 0.3), v_j = (0.6, 0.1, 0.3)$, and $q = 8$.

$$\begin{aligned}
&q-ROFHWA\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \S_{\overline{\ltimes}_{21}}, \S_{\overline{\ltimes}_{22}}\right) \\
&= \left(\sqrt[q]{1 - \prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1 - \frown_{\overline{\ltimes}_k}^{q}\right)^{w_i}\right)^{v_i}}, \prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}\right) \\
&= (0.732, 0.488).
\end{aligned} \tag{8}$$

Table 3: q-ROFHS values.

$(\hookleftarrow, \overline{\overline{\curlywedge}})$	y_1	y_2	y_3	y_4
$(\natural_{11}, \natural_{21}, \natural_{31})$	(.8, .9)	(.7, .9)	(.7, .8)	(.8, .9)
$(\natural_{11}, \natural_{21}, \natural_{32})$	(.7, .8)	(.6, .9)	(.7, .9)	(.7, .8)
$(\natural_{12}, \natural_{21}, \natural_{31})$	(.7, .9)	(.8, .9)	(.7, .8)	(.7, .9)
$(\natural_{12}, \natural_{21}, \natural_{32})$	(.8, .9)	(.7, .9)	(.7, .9)	(.8, .9)

Table 4: q-ROFHS values.

$(\looparrowleft, \overline{\beta})$	y_1	y_2	y_3	y_4
$(\natural_{11}, \natural_{21}, \natural_{31})$	(.9, .8)	(.9, .8)	(.8, .7)	(.9, .7)
$(\natural_{11}, \natural_{21}, \natural_{32})$	(.8, .7)	(.8, .7)	(.8, .7)	(.8, .7)
$(\natural_{12}, \natural_{21}, \natural_{31})$	(.8, .8)	(.9, .7)	(.9, .7)	(.8, .7)
$(\natural_{12}, \natural_{21}, \natural_{32})$	(.9, .8)	(.9, .8)	(.9, .8)	(.9, .8)

Theorem 1. *Let $\S_{\overline{\ltimes}_k} = (\frown_{\overline{\ltimes}_k}, \ddot{\frown}_{\overline{\ltimes}_k})$ be a q-ROFHN. Then, the aggregated result for $q-$ ROFHWA operator is given as*

$$q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) = \oplus_{j=1}^{\ddot{m}} v_i\left(\oplus_{i=1}^{\ddot{n}} w_i \S_{\overline{\ltimes}_{i_j}}\right)$$
$$= \left(\sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown_{\overline{\ltimes}_k}^{q}\right)^{w_i}\right)^{v_i}}, \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}\right), \tag{9}$$

where $w_i = \{w_1, w_2, \ldots, w_{\ddot{n}}\}$ and $v_i = \{v_1, v_2, \ldots, v_{\ddot{m}}\}$ are the expert weight vectors and selected subattributes, respectively, with given circumstances $w_i > 0, \sum_{i=1}^{\ddot{n}} w_i = 1, v_i > 0, \sum_{i=1}^{\ddot{m}} v_i = 1$.

Proof. Consider the principle of mathematical induction to verify the given results: for $\ddot{n} = 1$, we get $w_1 = 1$. Then, we have

$$q-\mathrm{ROFHWA}\left(I_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) = \oplus_{j=1}^{\ddot{m}} v_j \S_{\ltimes}$$
$$= \left(\sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(1-\frown_{\overline{\ltimes}_k}^{q}\right)^{v_i}}, \prod_{j=1}^{\ddot{m}}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{v_i}\right)$$
$$= \left(\sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{1}\left(1-\frown_{\overline{\ltimes}_k}^{q}\right)^{w_i}\right)^{v_i}}, \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{1}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}\right). \tag{10}$$

For $\ddot{m} = 1$, we get $v_1 = 1$. Then, we have

$$q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) = \oplus_{i=1}^{\ddot{n}} w_i \S_{\overline{\ltimes}_{i_j}}$$
$$= \left(\sqrt[q]{1-\prod_{i=1}^{\ddot{n}}\left(1-\frown_{\overline{\ltimes}_k}^{q}\right)^{w_i}}, \prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i}\right)$$
$$= \left(\sqrt[q]{1-\prod_{j=1}^{1}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown_{\overline{\ltimes}_k}^{q}\right)^{w_i}\right)^{v_j}}, \prod_{j=1}^{1}\left(\prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_j}\right). \tag{11}$$

This verifies that equation (4) is correct for $n = 1$ and $m = 1$. We will now show that equation (5) also holds for $\ddot{m} = 1$ and$\ddot{m} = 2$, thus completing our proof.

$$\begin{aligned}
q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) &= \oplus_{j=1}^{2} v_j\left(\oplus_{i=1}^{2} w_i \S_{\overline{\ltimes}_{i_j}}\right)\\
&= v_1\left(\oplus_{i=1}^{2} w_i \S_{\overline{\propto}_{i_1}}\right) \oplus v_2\left(\oplus_{i=1}^{2} w_i \S_{\overline{\propto}_{i_2}}\right)\\
&= v_1\left(w_1 \S_{\overline{\ltimes}_{11}} \oplus w_2 \S_{\overline{\ltimes}_{21}}\right) \oplus v_2\left(w_1 \S_{\overline{\ltimes}_{12}} \oplus w_2 \S_{\overline{\ltimes}_{22}}\right)\\
&= v_1\left\{\left(\sqrt[q]{1-(1-\frown_{11}^{q})^{w_1}}, \tilde{\frown}_{11}^{w_1}\right) \oplus \left(\sqrt[q]{1-(1-\frown_{21}^{q})^{w_2}}, \ddot{\frown}_{21}^{w_2}\right)\right\}\\
&\quad \oplus v_2\left\{\left(\sqrt[q]{1-(1-\frown_{12}^{q})^{w_1}}, \ddot{\frown}_{12}^{w_1}\right) \oplus \left(\sqrt[q]{1-(1-\frown_{22}^{q})^{w_2}}, \ddot{\frown}_{22}^{w_2}\right)\right\}\\
&= v_1\left(\sqrt[q]{1-\prod_{i=1}^{2}(1-\frown_{i1}^{q})^{w_i}}, \prod_{i=1}^{\ddot{n}} \ddot{\frown}_{i1}^{w_i}\right) \oplus v_2\left(\sqrt[q]{1-\prod_{i=1}^{2}(1-\frown_{i2}^{q})^{w_i}}, \prod_{i=1}^{\ddot{n}} \ddot{\frown}_{i2}^{w_i}\right)\\
&= \left(\sqrt[q]{1-\left(\prod_{i=1}^{2}(1-\frown_{i1}^{q})^{w_i}\right)^{v_1}}, \left(\prod_{i=1}^{2} \ddot{\frown}_{i1}^{w_i}\right)^{v_1}\right) \oplus \left(\begin{array}{c}\sqrt[q]{1-\left(\prod_{i=1}^{2}(1-\frown_{i2}^{q})^{w_i}\right)^{v_2}},\\ \left(\prod_{i=1}^{2} \ddot{\frown}_{i2}^{w_i}\right)^{v_2}\end{array}\right)\\
&= \left(\sqrt[q]{1-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}(1-\frown_{ij}^{q})^{w_i}\right)^{v_j}}, \prod_{j=1}^{2}\left(\prod_{i=1}^{2} \ddot{\frown}_{ij}^{w_i}\right)^{v_1}\right).
\end{aligned} \tag{12}$$

Hence, it is true for $\ddot{m} = k_1 + 1$ and $\ddot{n} = k_2 + 1$. Therefore, equation (28) is true for all $\ddot{m}, \ddot{n} \geq 1$, by mathematical induction. □

Theorem 2. *Let $\S_{\overline{\ltimes}_k} = (\frown_{\overline{\curlywedge}(\overline{\ltimes}_k)}, \ddot{\frown}_{\overline{\curlywedge}(\overline{\ltimes}_k)})$ be a q-ROFHN and $w_i = \{w_1, w_2, \ldots, w_{\ddot{n}}\}$ and $v_i = \{v_1, v_2, \ldots, v_{\ddot{m}}\}$ be the expert weight vectors and selected subattributes, having the* *condition that* $w_i > 0, \sum_{i=1}^{\ddot{n}} w_i = 1, v_i > 0, \sum_{i=1}^{\ddot{m}} v_i = 1$. *Then, the q-ROFHWA operator holds for the following properties:*

(i) *Idempotency: if* $\S_{\overline{\ltimes}_k} = \zeta_{\overline{\ltimes}}$ *for all* $(i = 1,2, \ldots, \ddot{n})$ *and* $(j = 1,2, \ldots, \ddot{m})$, *then q-ROFHWA* $(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}) = \zeta_{\overline{\ltimes}}$.

Proof. Since we know $\S_{\overline{\ltimes}_k} = (\frown_{\bar{\bar{\wedge}}(\propto\overline{\ltimes}_k)}, \ddot{\frown}_{\bar{\bar{\wedge}}(\overline{\ltimes}_k)}) = \zeta_{\overline{\ltimes}}$ is a collection of q-ROFHNs, then from Theorem 1, we have

$$
\begin{aligned}
& q - ROFHWA\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) \\
&= \left(\sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(1 - \frown_{\overline{\ltimes}_k}^{q}\right)^{w_i} \right)^{v_j}}, \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{w_i} \right)^{v_j} \right) \\
&= \left(\sqrt[q]{1 - \left(\left(1 - \frown_{\overline{\ltimes}_k}^{q}\right)^{\sum_{i=1}^{\ddot{n}} w_i} \right)^{\sum_{j=1}^{\ddot{m}} v_j}}, \left(\left(\ddot{\frown}_{\overline{\ltimes}_k}\right)^{\sum_{i=1}^{\ddot{n}} w_i} \right)^{\sum_{j=1}^{\ddot{m}} v_j} \right) \\
&= \left(\sqrt[q]{1 - \left(1 - \frown_{\overline{\ltimes}_k}^{q}\right)}, \tilde{\frown}_{\overline{\ltimes}_k} \right) \\
&= \zeta_{\overline{\ltimes}}.
\end{aligned}
\tag{13}
$$

Therefore,

$$
q - \mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) = \zeta_{\overline{\ltimes}}. \tag{14}
$$

(ii) Boundedness: if

$$
\S_{\overline{\ltimes}_{ij}}^{-} = \left(\min_j \min_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\}, \max_j \max_i \left\{ \ddot{\frown}_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\} \right), \tag{15}
$$

$$
\S_{\overline{\propto}_{ij}}^{+} = \left(\max_j \max_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\}, \min_j \min_i \left\{ \ddot{\frown}_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\} \right), \tag{16}
$$

then

$$
\S_{\overline{\ltimes}_{ij}}^{-} \leq q - \mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) \leq \S_{\overline{\ltimes}_{ij}}^{+}. \tag{17}
$$

□

Proof. $\S_{\overline{\ltimes}_{ij}}^{-} = (\min_j \min_i \{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \}, \max_j \max_i \{ \tilde{\frown}_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \})$ and $\S_{\overline{\ltimes}_{ij}}^{+} = (\max_j \max_i \{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \}, \min_j \min_i \{ \ddot{\frown}_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \})$. To prove that $\S_{\overline{\propto}_{ij}}^{-} \leq q\text{–ROFHWA}(\S_{\overline{\propto}_{11}}, \S_{\overline{\propto}_{12}}, \ldots, \S_{\overline{\propto}_{nm}}) \leq \S_{\overline{\propto}_{ij}}^{+}$, for each $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, we have

$$
\begin{aligned}
& \min_j \min_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\} \leq \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \leq \max_j \max_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})} \right\} \\
& \Rightarrow 1 - \max_j \max_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right\} \leq 1 - \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \\
& \leq 1 - \min_j \min_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right\} \\
& \Leftrightarrow \left(1 - \max_j \max_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right\} \right)^{w_i} \leq \left(1 - \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right)^{w_i} \\
& \leq \left(1 - \min_j \min_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right\} \right)^{w_i} \\
& \Leftrightarrow \left(1 - \max_j \max_i \left\{ \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right\} \right)^{\sum_{i=1}^{\ddot{n}} w_i} \leq \prod_{i=1}^{\ddot{n}} \left(1 - \frown_{\bar{\bar{\wedge}}(\overline{\ltimes}_{ij})}^{q} \right)^{w_i}
\end{aligned}
$$

$$
\begin{aligned}
&\leq \left(1-\min_j \min_i\left\{\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{\sum_{i=1}^{\ddot{n}} w_i}\\
&\Leftrightarrow \left(1-\max_j \max_i\left\{\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{\sum_{j=1}^{\ddot{m}} v_j} \leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}\\
&\leq \left(1-\min_j \min_i\left\{\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{\sum_{j=1}^{\ddot{m}} v_j}\\
&\Leftrightarrow 1-\max_j \max_i\left\{\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\} \leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}\\
&\leq 1-\min_j \min_i\left\{\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\\
&\Leftrightarrow \min_j \min_i\left\{\frown_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\} \leq 1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}\\
&\leq \max_j \max_i\left\{\frown_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}.
\end{aligned} \tag{18}
$$

Hence,

$$
\min_j \min_i\left\{\frown_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\} \leq \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\frown^{q}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}} \leq \max_j \max_i\left\{\frown_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}. \tag{19}
$$

Next, for each $i=1,2,\ldots,\ddot{n}$ and $j=1,2,\ldots,\ddot{m}$, we have

$$
\begin{aligned}
&\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\} \leq \ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right) \leq \max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\\
&\Leftrightarrow \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{w_i}\right)^{v_j} \leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}\\
&\leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{w_i}\right)^{v_j}\\
&\Leftrightarrow \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{\sum_{i=1}^{\ddot{n}} w_i}\right)^{\sum_{j=1}^{\ddot{m}} v_j} \leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j}\\
&\leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}\right)^{\sum_{i=1}^{\ddot{n}} w_i}\right)^{\sum_{j=1}^{\ddot{m}} v_j},
\end{aligned} \tag{20}
$$

which implies that

$$
\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\propto}_{ij}\right)\right\} \leq \prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right)^{w_i}\right)^{v_j} \leq \max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}}\left(\overline{\ltimes}_{ij}\right)\right\}. \tag{21}
$$

Therefore, $q-\mathrm{ROFHWA}(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{\ddot{n}\ddot{m}}}) = (\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}, \ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}) = \S_{\overline{\ltimes}_{ij}}$, and thus

$$\min_j \min_i\left\{\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\} \leq \sqrt[q]{\frown^q_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}} \leq \max_j \max_i\left\{\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\}. \tag{22}$$

$$\min_j \min_i\left\{\ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\} \leq \ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})} \leq \max_j \max_i\left\{\ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\}. \tag{23}$$

So, we have $S(\S_{\overline{\ltimes}_{ij}}) = (\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})})^q - (\ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})})^q \leq \max_j \max_i\left\{\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\}^q - \min_j \min_i\left\{\ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\}^q = S(\S^+_{\overline{\ltimes}_{ij}})$, $S(\S_{\overline{\ltimes}_{ij}}) = (\frown_{\overline{\overline{\wedge}}(\overline{\propto}_{ij})})^q - (\ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})})^q \geq \min_j \min_i\left\{\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_{ij})}\right\}^q - \max_j \max_i\left\{\ddot{\frown}_{\overline{\ltimes}(\overline{\ltimes}_{ij})}\right\}^q = S(\S^-_{\overline{\ltimes}_{ij}})$.
Then,

$$S\left(\S^-_{\overline{\ltimes}_{ij}}\right) \leq q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\propto}_{\ddot{n}\ddot{m}}}\right) \leq S\left(\S^+_{\overline{\ltimes}_{ij}}\right). \tag{24}$$

□

Definition 8. Let $\S_{\overline{\ltimes}_k} = (\frown_{\overline{\overline{\wedge}}(\overline{\ltimes}_k)}, \ddot{\frown}_{\overline{\overline{\wedge}}(\overline{\ltimes}_k)})$ be a q-ROFHN, $w_i = \{w_1, w_2, \ldots, w_{\ddot{n}}\}$ and $v_i = \{v_1, v_2, \ldots, v_{\ddot{m}}\}$ be the weight vectors of the experts and selected parameters of subattributes, respectively, having the condition that $w_i > 0, \sum_{i=1}^{\ddot{n}} w_i = 1, v_i > 0, \sum_{i=1}^{\ddot{m}} v_i = 1$. Then, the mapping for q-ROFHWG operator is defined as $q-\mathrm{ROFHWG}: \Delta^{\ddot{n}} \longrightarrow \Delta$, where Δ is the collection of all q-ROFHNs.

$$q-\mathrm{ROFHWG}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{\ddot{n}\ddot{m}}}\right) = \otimes_{j=1}^{\ddot{m}} v_i\left(\otimes_{i=1}^{\ddot{n}} w_i \S_{\overline{\ltimes}_{i_j}}\right). \tag{25}$$

Example 4. Let Ⓢ be the set of decision makers to decide the best car given as Ⓢ $= \{y_1, y_2, y_3\}$ and also consider the set of attributes as $\urcorner_1$ and $\urcorner_2$, where $\urcorner_1$ represents colour of the car and $\urcorner_2$ represents price of the car. Then, their corresponding attributive sets can be $\urcorner_1 = \{a_{11} = \text{black}, a_{12} = \text{white}\}$, $\urcorner_2 = \{a_{21} = 25\text{lac}, a_{22} = 30\text{lac}, a_{23} = 20\text{laclac}\}$.

Suppose $\overline{\wedge}_1 = \{\natural_{11}, \natural_{12}\}, \overline{\wedge}_2 = \{\natural_{21}, \natural_{22}\}$. Then, $\overline{\overline{\wedge}} = \urcorner_1 \times \urcorner_2$ will have 2 tuple elements and we suppose $q = 8$.
Then, q-ROFHS set $(\looparrowleft, \overline{\overline{\wedge}})$ can be written as

$$\begin{aligned}\langle \S_{\overline{\ltimes}_{11}}\rangle &= \langle 0.91, 0.84\rangle,\\ \langle \S_{\overline{\ltimes}_{12}}\rangle &= \langle 0.85, 0.92\rangle,\\ \langle \S_{\overline{\ltimes}_{21}}\rangle &= \langle 0.80, 0.93\rangle,\\ \langle \S_{\overline{\ltimes}_{22}}\rangle &= \langle 0.95, 0.88\rangle.\end{aligned} \tag{26}$$

Let $w_i = (0.2, 0.1, 0.4, 0.3), v_j = (0.6, 0.1, 0.3)$, and $q = 8$.

$$\begin{aligned}&q-ROFHWG\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \S_{\overline{\ltimes}_{21}}, \S_{\overline{\ltimes}_{11}}\right)\\ &= \left(\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(\frown_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}, \sqrt[q]{1-\prod_{j=1}^{3}\left(\prod_{i=1}^{4}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}}\right)\\ &= (0.895, 0.569).\end{aligned} \tag{27}$$

Theorem 3. *Let $\S_{\overline{\ltimes}_k} = (\frown_{\overline{\ltimes}_k}, \ddot{\frown}_{\overline{\ltimes}_k})$ be a q-ROFHN. Then, the aggregated result for $q-\mathrm{ROFHWG}$ operator is given as*

$$\begin{aligned}q-\mathrm{ROFHWG}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) &= \otimes_{j=1}^{\ddot{m}} v_i\left(\otimes_{i=1}^{\ddot{n}} w_i \S_{\overline{\ltimes}_{i_j}}\right)\\ &= \left(\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\frown_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}, \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}}\right),\end{aligned} \tag{28}$$

where $w_i = \{w_1, w_2, \ldots, w_{\ddot{n}}\}$ and $v_i = \{v_1, v_2, \ldots, v_{\ddot{m}}\}$ are the weight vectors of the experts and selected parameters of subattributes, respectively, with given circumstances $w_i > 0, \sum_{i=1}^{\ddot{n}} w_i = 1, v_i > 0, \sum_{i=1}^{\ddot{m}} v_i = 1$.

Proof of Theorem 3. Consider the principle of mathematical induction to verify the given result as follows: for $\ddot{n} = 1$, we get $w_1 = 1$. Then, we have

$$\begin{aligned}q-\mathrm{ROFHWG}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) &= \otimes_{j=1}^{\ddot{m}} \S^{v_j}_{\overline{\ltimes}_{i_j}}\\ &= \left(\prod_{j=1}^{\ddot{m}}\left(\frown_{\overline{\ltimes}_k}\right)^{v_i}, \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{v_i}}\right)\\ &= \left(\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{1}\left(\frown_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}, \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{1}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_i}}\right).\end{aligned} \tag{29}$$

For $\ddot{m} = 1$, we get $v_1 = 1$. Then, we have

$$\begin{aligned}q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right) &= \otimes_{i=1}^{\ddot{n}}\left(\S_{\overline{\ltimes}_{i_1}}\right)^{w_i}\\ &= \left(\prod_{i=1}^{\ddot{n}}\left(\frown_{\overline{\ltimes}_k}\right)^{w_i}, \sqrt[q]{1-\prod_{i=1}^{\ddot{n}}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{w_i}}\right)\\ &= \left(\prod_{j=1}^{1}\left(\prod_{i=1}^{\ddot{n}}\left(\frown_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{1}\left(\prod_{i=1}^{\ddot{n}}\left(1-\ddot{\frown}^q_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_j}}\right).\end{aligned} \tag{30}$$

For $\ddot{n} = 1$ and $\ddot{m} = 2$, this proves that equation (28) is true. Now we will prove that equation (28) also holds for $\ddot{n} = 2$ and $\ddot{m} = 1$, so we have

$$
\begin{aligned}
&q-\mathrm{ROFHWA}\left(\S_{\overline{\ltimes}_{11}}, \S_{\overline{\ltimes}_{12}}, \ldots, \S_{\overline{\ltimes}_{nm}}\right)=\oplus_{j=1}^{2} v_j\left(\otimes_{i=1}^{2} w_i \S_{\overline{\ltimes}_{i_j}}\right)\\
&\quad= v_1\left(\oplus_{i=1}^{2} w_i \S_{\overline{\ltimes}_{i_1}}\right)\otimes v_2\left(\oplus_{i=1}^{2} w_i \S_{\overline{\ltimes}_{i_2}}\right)=v_1\left(w_1\S_{\overline{\ltimes}_{11}}\otimes w_2\S_{\overline{\ltimes}_{21}}\right)\otimes v_2\left(w_1\S_{\overline{\ltimes}_{12}}\otimes w_2\S_{\overline{\ltimes}_{22}}\right)\\
&\quad= v_1\left\{\left(\frown_{11}^{w_1}, \sqrt[q]{1-\left(1-\ddot{\frown}_{11}^{q}\right)^{w_1}}\right)\otimes\left(\frown_{21}^{w_2}, \sqrt[q]{1-\left(1-\ddot{\frown}_{21}^{q}\right)^{w_2}}\right)\right\}\otimes v_2\left\{\left(\frown_{12}^{w_1}, \sqrt[q]{1-\left(1-\tilde{\frown}_{12}^{q}\right)^{w_1}}\right)\otimes\left(\frown_{22}^{w_2}, \sqrt[q]{1-\left(1-\ddot{\frown}_{22}^{q}\right)^{w_2}}\right)\right\}\\
&\quad= v_1\left(\prod_{i=1}^{\ddot{n}}\frown_{i1}^{w_i}, \sqrt[q]{1-\prod_{i=1}^{2}\left(1-\ddot{\frown}_{i1}^{q}\right)^{w_i}}\right)\otimes v_2\left(\prod_{i=1}^{\ddot{n}}\frown_{i1}^{w_i}, \sqrt[q]{1-\prod_{i=1}^{2}\left(1-\ddot{\frown}_{i2}^{q}\right)^{w_i}}\right)\\
&\quad=\left(\left(\prod_{i=1}^{2}\frown_{i1}^{w_i}\right)^{v_1}, \sqrt[q]{1-\left(\prod_{i=1}^{2}\left(1-\ddot{\frown}_{i1}^{q}\right)^{w_i}\right)^{v_1}}\right)\otimes\left(\begin{array}{c}\left(\prod_{i=1}^{2}\frown_{i2}^{w_i}\right)^{v_2}\\ \sqrt[q]{1-\left(\prod_{i=1}^{2}\left(1-\ddot{\frown}_{i2}^{q}\right)^{w_i}\right)^{v_2}}\end{array}\right)\\
&\quad=\left(\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\frown_{ij}^{w_i}\right)^{v_1}, \sqrt[q]{1-\prod_{j=1}^{2}\left(\prod_{i=1}^{2}\left(1-\ddot{\frown}_{ij}^{q}\right)^{w_i}\right)^{v_j}}\right).
\end{aligned}
\tag{31}
$$

Hence, the result is true for $\ddot{n}=2$ and $\ddot{m}=2$. Further suppose that equation (28) is true for $\ddot{m}=k_1+1, \ddot{n}=k_2$ and $\ddot{m}=k_1, \ddot{n}=k_2+1$, such as

$$
\begin{aligned}
&\otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2} w_i \S_{\overline{\ltimes}_{ij}}\right)=\left(\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2}\frown_{ij}^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2}\left(1-\ddot{\frown}_{i1}^{q}\right)^{w_i}\right)^{v_j}}\right),\\
&\otimes_{j=1}^{k_1} v_j\left(\otimes_{i=1}^{k_2+1} w_i \S_{\overline{\ltimes}_{ij}}\right)=\left(\prod_{j=1}^{k_1}\left(\prod_{i=1ij}^{k_2+1}\frown^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{k_1}\left(\prod_{i=1}^{k_2+1}\left(1-\ddot{\frown}_{i1}^{q}\right)^{w_i}\right)^{v_j}}\right).
\end{aligned}
\tag{32}
$$

For $\ddot{m}=k_1+1, \ddot{n}=k_2+1$, we have

$$
\begin{aligned}
&\otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2+1} w_i \S_{\overline{\ltimes}_{ij}}\right)=\otimes_{j=1}^{k_1+1} v_j\left(\otimes_{i=1}^{k_2} w_i \S_{\overline{\ltimes}_{ij}}\otimes w_{k_2+1}\S_{\overline{\ltimes}_{(k_2+1)j}}\right)\\
&\quad=\otimes_{j=1}^{k_1+1}\otimes_{i=1}^{k_2} v_j w_i \S_{\overline{\ltimes}_{ij}}\otimes_{j=1}^{k_1+1} v_j w_{k_2+1}\S_{\overline{\ltimes}_{(k_2+1)j}}\\
&\quad=\left(\begin{array}{c}\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2}\frown_{ij}^{w_i}\right)^{v_j}\otimes\prod_{j=1}^{k_1+1}\left(\left(\frown_{\overline{\ltimes}(k_2+1)j}\right)^{w_{k_2+1}}\right)^{v_j}\\ \sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2}\left(1-\ddot{\frown}_{i1}^{q}\right)^{w_i}\right)^{v_j}}\otimes\sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\left(1-\ddot{\frown}_{\overline{\ltimes}(k_2+1)j}^{q}\right)^{w_{k_2+1}}\right)^{v_j}}\end{array}\right)\\
&\quad=\left(\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\frown_{ij}^{w_i}\right)^{v_j}, \sqrt[q]{1-\prod_{j=1}^{k_1+1}\left(\prod_{i=1}^{k_2+1}\left(1-\ddot{\frown}_{\overline{\ltimes}(k_2+1)j}^{q}\right)^{w_i}\right)^{v_j}}\right).
\end{aligned}
\tag{33}
$$

Hence, it is true for $\ddot{m}=k_1+1$ and $\ddot{n}=k_2+1$. Therefore, equation (26) is true for all $\ddot{m},\ddot{n}\geq 1$, by mathematical induction. □

Theorem 4. *Let* $\S_{\overline{\ltimes}_k}=(\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_k)},\ddot{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_k)})$ *be a q-ROFHN and* $w_i=\{w_1,w_2,\ldots,w_{\ddot{n}}\}$ *and* $v_i=\{v_1,v_2,\ldots,v_{\ddot{m}}\}$ *be the weight vectors of the experts and selected parameters of subattributes, respectively, having the condition that* $w_i>0,\sum_{i=1}^{\ddot{n}}w_i=1,v_i>0,\sum_{i=1}^{\ddot{m}}v_i=1$. *Then, the q-ROFHWA operator holds for the following properties:*

Idempotency: if $\S_{\overline{\ltimes}_k}=\zeta_{\overline{\ltimes}}$ *for all (i = 1, 2, . . ., n) and (j = 1, 2, . . ., m), then q-ROFHWA* $(\S_{\overline{\ltimes}_{11}},\S_{\overline{\ltimes}_{12}},\ldots,\S_{\overline{\ltimes}_{nm}})=\zeta_{\overline{\ltimes}}$.

Proof. As we know $\S_{\overline{\ltimes}_k}=(\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_k)},\ddot{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_k)})=\zeta_{\overline{\ltimes}}$ is a collection of q-ROFHNs, then from Theorem 1, we have

$$\begin{aligned}
q-ROFHWA\left(\S_{\overline{\ltimes}_{11}},\S_{\overline{\ltimes}_{12}},\ldots,\S_{\overline{\ltimes}_{nm}}\right)&=\left(\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\curvearrowright_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_j},\sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\ddot{\curvearrowright}^{q}_{\overline{\ltimes}_k}\right)^{w_i}\right)^{v_j}}\right)\\
&=\left(\left(\left(\curvearrowright_{\overline{\ltimes}_k}\right)^{\sum_{i=1}^{\ddot{n}}w_i}\right)^{\sum_{j=1}^{\ddot{m}}v_j},\sqrt[q]{1-\left(\left(1-\ddot{\curvearrowright}^{q}_{\overline{\ltimes}_k}\right)^{\sum_{i=1}^{\ddot{n}}w_i}\right)^{\sum_{j=1}^{\ddot{m}}v_j}}\right)\\
&=\left(\curvearrowright_{\overline{\ltimes}_k},\sqrt[q]{1-\left(1-\ddot{\curvearrowright}^{q}_{\overline{\ltimes}_k}\right)}\right)\\
&=\zeta_{\overline{\ltimes}}.
\end{aligned}\tag{34}$$

Therefore,

$$q-\mathrm{ROFHWG}\left(\S_{\overline{\ltimes}_{11}},\S_{\overline{\ltimes}_{12}},\ldots,\S_{\overline{\ltimes}_{nm}}\right)=\zeta_{\overline{\ltimes}}.\tag{35}$$

(ii) Boundedness: if

$$\S^{-}_{\overline{\ltimes}_{ij}}=\left(\min_j\min_i\left\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\},\max_j\max_i\left\{\ddot{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right),\tag{36}$$

$$\S^{+}_{\overline{\ltimes}_{ij}}=\left(\max_j\max_i\left\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\},\min_j\min_i\left\{\ddot{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right),\tag{37}$$

then

$$\S^{-}_{\overline{\ltimes}_{ij}}\leq q-\mathrm{ROFHWG}\left(\S_{\overline{\ltimes}_{11}},\S_{\overline{\ltimes}_{12}},\ldots,\S_{\overline{\ltimes}_{nm}}\right)\leq\S^{+}_{\overline{\ltimes}_{ij}}.\tag{38}$$

□

Proof of Theorem 4. Part ii. $\S^{-}_{\overline{\ltimes}_{ij}}=(\min_j\min_i\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\},\max_j\max_i\{\tilde{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\})$ and $\S^{+}_{\overline{\ltimes}_{ij}}=(\max_j\max_i\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\},\min_j\min_i\{\ddot{\curvearrowright}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\})$. To prove that $\S_{\overline{\propto}_{ij}}{}^{-}\leq q\text{–ROFHWG}(\S_{\overline{\propto}_{11}},\S_{\overline{\propto}_{12}},\ldots,\S_{\overline{\propto}_{\ddot{n}\ddot{m}}})\leq\S_{\overline{\propto}_{ij}}{}^{+}$, for each $i=1,2,\ldots,\ddot{n}$ and $j=1,2,\ldots,\ddot{m}$, we have

$$\begin{aligned}
&\min_j\min_i\left\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\leq\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\\
&\leq\max_j\max_i\left\{\curvearrowright_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\\
&\Rightarrow\min_j\min_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\leq\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\leq\max_j\max_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\\
&\Leftrightarrow\left(\min_j\min_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right)^{w_i}\leq\left(\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right)^{w_i}\\
&\leq\left(\max_j\max_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right)^{w_i}\\
&\Leftrightarrow\left(\min_j\min_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right)^{\sum_{i=1}^{\ddot{n}}w_i}\leq\prod_{i=1}^{\ddot{n}}\left(\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right)^{w_i}\\
&\leq\left(\max_j\max_i\left\{\curvearrowright^{q}_{\overline{\barwedge}(\overline{\ltimes}_{ij})}\right\}\right)^{\sum_{i=1}^{\ddot{n}}w_i}
\end{aligned}$$

$$
\begin{aligned}
&\Leftrightarrow \left(\min_j \min_i \left\{ \curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \right)^{\sum_{j=1}^{\ddot{m}} v_j} \leq \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j} \\
&\leq \left(\max_j \max_i \left\{ \curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \right)^{\sum_{j=1}^{\ddot{m}} v_j} \\
&\Leftrightarrow \min_j \min_i \left\{ \curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \leq \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j} \\
&\leq \max_j \max_i \left\{ \curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \\
&\Leftrightarrow \min_j \min_i \left\{ \curvearrowright_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \leq \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(1 - \curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j} \\
&\leq \max_j \max_i \left\{ \curvearrowright_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\}.
\end{aligned} \tag{39}
$$

Hence,

$$
\min_j \min_i \left\{ \curvearrowright_{\doublebarwedge}\left(\overline{\propto}_{ij}\right) \right\} \leq \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\curvearrowright^{q}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j} \leq \max_j \max_i \left\{ \curvearrowright_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\}. \tag{40}
$$

Next, for each $i = 1,2, \ldots, \ddot{n}$ and $j = 1,2, \ldots, \ddot{m}$, we have

$$
\begin{aligned}
&\min_j \min_i \left\{ \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \leq \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \\
&\quad \leq \max_j \max_i \left\{ \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \\
&\quad \Leftrightarrow \sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\min_j \min_i \left\{ 1 - \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(1 - \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j}} \\
&\quad \leq \sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\max_j \max_i \left\{ 1 - \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \right)^{w_i} \right)^{v_j}} \\
&\quad \Leftrightarrow \sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(\min_j \min_i \left\{ 1 - \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right\} \right)^{\sum_{i=1}^{\ddot{n}} w_i} \right)^{\sum_{j=1}^{\ddot{m}} v_j}} \leq \sqrt[q]{1 - \prod_{j=1}^{\ddot{m}} \left(\prod_{i=1}^{\ddot{n}} \left(1 - \ddot{\curvearrowright}_{\doublebarwedge}\left(\overline{\ltimes}_{ij}\right) \right)^{w_i} \right)^{v_j}}
\end{aligned}
$$

$$\leq \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(\max_j \max_i\left\{1-\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}\right)^{\sum_{i=1}^{\ddot{n}} w_i}\right)^{\sum_{j=1}^{\ddot{m}} v_j}}, \tag{41}$$

which implies that

$$\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\} \leq \sqrt[q]{1-\prod_{j=1}^{\ddot{m}}\left(\prod_{i=1}^{\ddot{n}}\left(1-\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right)^{w_i}\right)^{v_j}} \leq \max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}. \tag{42}$$

Therefore, $q-\text{ROFHWG}\left(\S_{\bar{\ltimes}_{11}}, \S_{\bar{\ltimes}_{12}}, \ldots, \S_{\bar{\ltimes}_{\ddot{n}\ddot{m}}}\right) = \left(\frown_{\bar{\bar{\wedge}}(\bar{\ltimes}_{ij})}, \ddot{\frown}_{\bar{\bar{\wedge}}(\bar{\ltimes}_{ij})}\right) = \S_{\bar{\ltimes}_{ij}}$, so

$$\min_j \min_i\left\{\frown_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\} \leq \sqrt[q]{\frown^q_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}} \leq \max_j \max_i\left\{\frown_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}, \tag{43}$$

$$\min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\} \leq \ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)} \leq \max_j \max_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}. \tag{44}$$

So, we have

$$\begin{aligned}
S\left(\S_{\bar{\ltimes}_{ij}}\right) &= \left(\frown_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right)^q - \left(\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right)^q \\
&\leq \max_j \max_i\left\{\frown_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}^q - \min_j \min_i\left\{\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}^q \\
&= S\left(\S^{+}_{\bar{\ltimes}_{ij}}\right), \\
S\left(\S_{\bar{\ltimes}_{ij}}\right) &= \left(\frown_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right)^q - \left(\ddot{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right)^q \\
&\geq \min_j \min_i\left\{\frown_{\bar{\bar{\wedge}}\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}^q - \max_j \max_i\left\{\tilde{\frown}_{\bar{\bar{\wedge}}\left(\bar{\ltimes}_{ij}\right)}\right\}^q \\
&= S\left(\S^{-}_{\bar{\ltimes}_{ij}}\right).
\end{aligned} \tag{45}$$

Then,

$$S\left(\S^{-}_{\bar{\ltimes}_{ij}}\right) \leq q-\text{ROFHWG}\left(\S_{\bar{\ltimes}_{11}}, \S_{\bar{\ltimes}_{12}}, \ldots, \S_{\bar{\ltimes}_{\ddot{n}\ddot{m}}}\right) \leq S\left(\S^{+}_{\bar{\ltimes}_{ij}}\right). \tag{46}$$

□

4. Proposed Methodology

Decision making is a technique used to choose logical alternatives in different cases/environment. Here, we will use our proposed technique under q rung orthopair fuzzy hypersoft sets environment for multi criteria decision making process. Let $\{Q^1, Q^2, \ldots, Q^s\}$ be a set of s attributes and $\{x_1, x_2, \ldots, x_{\ddot{n}}\}$ be a set of n experts. The weight of experts are given as $w = \{w_1, w_2, \ldots, w_{\ddot{n}}\}^T$. Let $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_{\ddot{m}}\}$ be the set of attributes with their corresponding subattributes as $\varpi = \left\{\varpi_{1a} \times \varpi_{2a} \times, \ldots \times \varpi_{mp}\right\}$ for all $a \in \{1, 2, \ldots, t\}$ with weights $v = \left\{v_{1a} \times v_{2a} \times, \ldots \times v_{\ddot{m}p}\right\}$ such as $v_a > 0, \sum_{p=1}^{t} a_a = 1$. The following algorithm can be used to make a decision:

Step 1. Construct a decision matrix containing subattributes of parameters.

Step 2. Construct a decision matrix based on the experts' evaluations of each alternative in the form of q-ROFNs.

$$\begin{aligned}
(Q, \acute{\varpi}) &= \left[\S^q_{\bar{\ltimes}_{ij}}\right]_{\ddot{n}\times\ddot{m}} \\
&= \begin{pmatrix}
\S^q_{\bar{\ltimes}_{11}} & \S^q_{\bar{\ltimes}_{12}} & \cdots & \S^q_{\bar{\ltimes}_{1m}} \\
\S^q_{\bar{\ltimes}_{21}} & \S^q_{\bar{\ltimes}_{22}} & \cdots & \S^q_{\bar{\ltimes}_{2\ddot{m}}} \\
. & . & \cdots & . \\
. & . & \cdots & . \\
. & . & \cdots & . \\
\S^q_{\bar{\ltimes}_{\ddot{n}1}} & \S^q_{\bar{\ltimes}_{\ddot{n}2}} & \cdots & \S^q_{\bar{\ltimes}_{\ddot{n}\ddot{m}}}
\end{pmatrix}.
\end{aligned} \tag{47}$$

Figure 1: 1BTC = 46004.15 USD (source: price index data from CoinDesk (https://coinmarketcap.com/currencies/bitcoin/?period=7d)).

Table 5: Decision matrix for cryptocurrency market.

A_1	$\overline{\ltimes}_1$	$\overline{\ltimes}_2$	$\overline{\ltimes}_3$	$\overline{\ltimes}_4$	$\overline{\ltimes}_5$	$\overline{\ltimes}_6$	$\overline{\ltimes}_7$	$\overline{\ltimes}_8$
X_1	(.91, .97)	(.81, .97)	(.96, .98)	(.79, .68)	(.81, .98)	(.79, .98)	(.79, .96)	(.97, .98)
X_2	(.87, .98)	(.98, .88)	(.88, .98)	(.85, .99)	(.87, .99)	(.86, .87)	(.86, .97)	(.90, .98)
X_3	(.87, .89)	(.98, .79)	(.77, .99)	(.88, .97)	(.88, .89)	(.97, .99)	(.79, .98)	(.89, .79)
X_4	(.67, .98)	(.89, .98)	(.87, .99)	(.95, .86)	(.87, .96)	(.78, .89)	(.97, .87)	(.79, .98)

Table 6: Decision matrix for alternative (A_1 = Tether).

A_1	$\overline{\ltimes}_1$	$\overline{\ltimes}_2$	$\overline{\ltimes}_3$	$\overline{\ltimes}_4$	$\overline{\ltimes}_5$	$\overline{\ltimes}_6$	$\overline{\ltimes}_7$	$\overline{\ltimes}_8$
X_1	(.71, .98)	(.81, .91)	(.91, 0.98)	(.79, .68)	(.81, .98)	(.77, .98)	(.73, .94)	(.79, .98)
X_2	(.81, .88)	(.78, .88)	(.88, .98)	(.85, .99)	(.85, .89)	(.83, .88)	(.83, .87)	(.99, .78)
X_3	(.78, .89)	(.98, .69)	(.77, .69)	(.88, .77)	(.88, .89)	(.98, .89)	(.79, .98)	(.88, .79)
X_4	(.67, .87)	(.69, .88)	(.87, .99)	(.97, .86)	(.87, .96)	(.68, .89)	(.97, .87)	(.69, .88)

Table 7: Decision matrix for alternative (A_2 = Binance Coin (BNB)).

A_1	$\overline{\ltimes}_1$	$\overline{\ltimes}_2$	$\overline{\ltimes}_3$	$\overline{\ltimes}_4$	$\overline{\ltimes}_5$	$\overline{\ltimes}_6$	$\overline{\ltimes}_7$	$\overline{\ltimes}_8$
X_1	(.74, .98)	(.76, .98)	(.95, .98)	(.81, .91)	(.89, .98)	(.76, .98)	(.83, .94)	(.95, .93)
X_2	(.83, .87)	(.98, .88)	(.98, .88)	(.78, .88)	(.84, .99)	(.93, .88)	(.87, .85)	(.97, .76)
X_3	(.78, .79)	(.85, .89)	(.74, .79)	(.98, .69)	(.85, .89)	(.95, .89)	(.83, .89)	(.89, .74)
X_4	(.97, .87)	(.89, .76)	(.87, .93)	(.69, .88)	(.98, .79)	(.98, .84)	(.94, .86)	(.79, .84)

Step 3. Aggregate all the alternatives according to the proposed aggregation operators, i.e., weighted average and weighted geometric operator.

Step 4. Calculate each alternative's score.

Step 5. Choose the best option by ranking the alternatives according to the descending values of the score value.

4.1. Numerical Example. To demonstrate how each stage of the typical decision-making approach works, we present a practical stepwise procedure based on the following scenario. We want to examine the price stability of specific cryptocurrencies (alternative), denoted as A_1 = Tether, A_2 = Binance Coin, A_3 = Ethereum, and A_4 = Bitcoin. A committee of decision makers having weight vector (.16, .25, .33, .26) decides the best cryptocurrency. The attribute-valued sets ϖ = {security, decentralization, demand} with their corresponding subattributes are given as ϖ_1 = security = {a_{11} = strong level of security, a_{12} = low level of security}, ϖ_2 = decentralization = {a_{21} decentralized application (dApp), a_{22} = decentralized autonomous organization (DAO)}, and ϖ_3 = demand = {more demands, less demand}. Then, $\varpi = \varpi_1 \times \varpi_2 \times \varpi_3$ is a set of subattributes which have 3-tuple elements.

$$\begin{aligned}\acute{\varpi} &= \{\natural_{11}, \natural_{12}\} \times \{\natural_{21}, \natural_{22}\} \times \{\natural_{31}, \natural_{32}\} \\ &= \left\{\begin{array}{c} c(\natural_{11}, \natural_{21}, \natural_{31}), (\natural_{11}, \natural_{21}, \natural_{32}), (\natural_{11}, \natural_{22}, \natural_{31}), (\natural_{11}, \natural_{22}, \natural_{32}) \\ (\natural_{12}, \natural_{21}, \natural_{31}), (\natural_{12}, \natural_{21}, \natural_{32}), (\natural_{12}, \natural_{22}, \natural_{31}), (\natural_{12}, \natural_{22}, \natural_{32}) \end{array}\right\}. \end{aligned} \tag{48}$$

Table 8: Decision matrix for alternative (A_3 = Ethereum).

A_1	$\overline{\aleph}_1$	$\overline{\aleph}_2$	$\overline{\aleph}_3$	$\overline{\aleph}_4$	$\overline{\aleph}_5$	$\overline{\aleph}_6$	$\overline{\aleph}_7$	$\overline{\aleph}_8$
X_1	(.78, .94)	(.76, .98)	(.95, .98)	(.84, .95)	(.93, .84)	(.76, .94)	(.86, .97)	(.89, .98)
X_2	(.86, .87)	(.85, .89)	(.98, .99)	(.87, .87)	(.77, .65)	(.93, .98)	(.88, .85)	(.84, .99)
X_3	(.78, .77)	(.85, .89)	(.77, .78)	(.95, .79)	(.87, .88)	(.92, .89)	(.86, .89)	(.84, .99)
X_4	(.97, .88)	(.83, .79)	(.83, .97)	(.69, .88)	(.74, .89)	(.91, .88)	(.91, .86)	(.85, .89)

Table 9: Decision matrix for alternative (A_4 = Bitcoin).

A_1	$\overline{\aleph}_1$	$\overline{\aleph}_2$	$\overline{\aleph}_3$	$\overline{\aleph}_4$	$\overline{\aleph}_5$	$\overline{\aleph}_6$	$\overline{\aleph}_7$	$\overline{\aleph}_8$
X_1	(.79, .98)	(.79, .68)	(.95, .92)	(.83, .91)	(.89, .98)	(.76, .98)	(.83, .94)	(.95, .94)
X_2	(.83, .84)	(.88, .69)	(.98, .76)	(.78, .81)	(.84, .99)	(.93, .88)	(.86, .81)	(.97, .86)
X_3	(.88, .72)	(.95, .78)	(.84, .78)	(.98, .91)	(.85, .89)	(.95, .89)	(.88, .81)	(.81, .84)
X_4	(.99, .88)	(.69, .97)	(.89, .73)	(.71, .88)	(.98, .79)	(.98, .84)	(.91, .88)	(.71, .88)

Figure 2: 1ETH = 3760.10 USD (source: price index data from CoinDesk (https://coinmarketcap.com/currencies/ethereum/?period=7d)).

Figure 3: 1BNB = 508.32 USD (https://coinmarketcap.com/currencies/bnb/?period=7d).

Let $\acute{\varpi} = \{\overline{\aleph}_1, \overline{\aleph}_2, \overline{\aleph}_3, \overline{\aleph}_4, \overline{\aleph}_5, \overline{\aleph}_6, \overline{\aleph}_7, \overline{\aleph}_8\}$ be a set of all subattributes with (.22,.1,.04,.07,.13,.14.11,.17) weights. Under the examined subattributes, each expert will assess each alternative's ratings in the form of q-ROFHNs. The following are the decision processes to determine the optimal alternative using the q-ROFHWA or q-ROFHWG operators:

Step 1. Develop a matrix containing subattributes of parameters (see Table 5).

Figure 4: 1Tether = 1.00 USD (https://coinmarketcap.com/currencies/tether/?period=7d).

Step 2. Develop the decision matrices (see Tables 6–9) of the experts as follows.

Step 3. Aggregated values of experts from given tables are calculated using the given weighted average aggregation operator as follows:

$$\begin{aligned} &A_1, (.881576, .869115),\\ &A_2, (.879565, .860845),\\ &A_3, (.907279, .881586),\\ &A_4, (.916721, 841755). \end{aligned} \tag{49}$$

Step 4. Score value of alternatives.

$$\begin{aligned} A_1 &= 0.012461,\\ A_2 &= 0.01872,\\ A_3 &= 0.025693,\\ A_4 &= 0.074966. \end{aligned} \tag{50}$$

Step 5. The rank of alternatives shows us that $A_4 > A_3 > A_2 > A_1$, and Bitcoin (alternative A_4) is the best among all these cryptocurrencies (alternatives). According to the team of experts, a Bitcoin has a strong security level and decentralized application. Also, Bitcoin has more market demand as compared to other

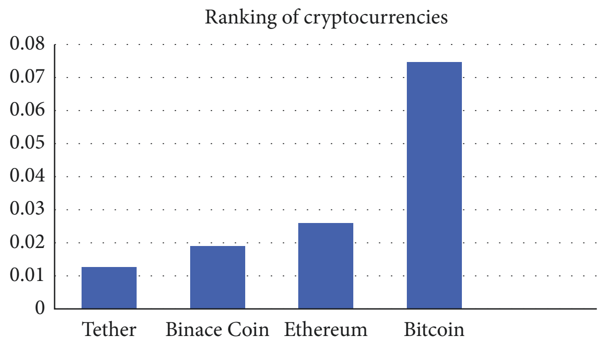

Figure 5: Overall ranking of cryptocurrency.

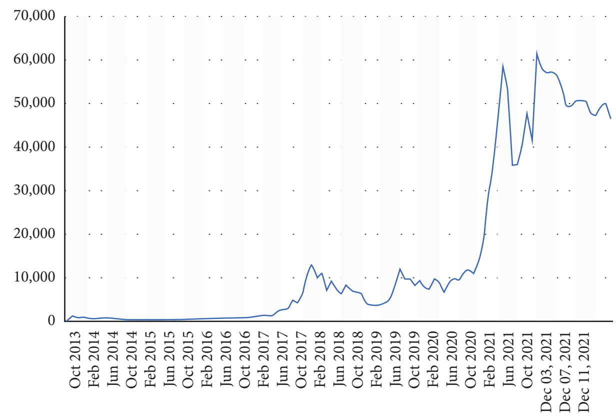

Figure 6: Bitcoin (BTC) price from 2013 to 2021 (source: price index data from CoinDesk (http://www.coindesk.com/price/)).

cryptocurrencies because Bitcoin is a good indicator of the crypto market. Bitcoin's price has rapidly grown in recent months. The latest price rise in the world's most popular cryptocurrency has generated speculation about its future. Therefore, the team of experts announced its decision on Bitcoin. Graphical representation of cryptocurrency (alternatives) is shown in Figure 1.

As we have seen from the result, Bitcoin has a high range compared to other cryptocurrencies. These all have a different price validity in relation to time. First of all, we will discuss the volatility of Bitcoin prices as in April 2011, from 1 USD to a peak of 29.60 USD by June. 2012 was a rather quiet year for Bitcoin, while 2013 saw significant price increases. Bitcoin trading started in January 2013 at 13.28 and peaked at 230 in April. In 2014, its price reached 315.21 USD. Also, its prices still have a big increase from 900 USD in 2016 to 40,000 USD in 2021. The graphical representation of Bitcoin, Ethereum, Binance Coin, and Tether is presented in Figures 2–5.

The combined graphical representation of the price of Bitcoin, Ethereum, Binance Coin, and Tether is shown in Figure 6.

5. Comparison Analysis

We will compare our suggested structure to the present one in this section. Our proposed structure has multiple choices of attributes in which we deal with uncertainty more generally with respect to the fuzzy soft set theory [21], intuitionistic fuzzy soft set [33], the Pythagorean fuzzy soft set [34], and the q-rung orthopair fuzzy soft set [35]. All these existing structures are widely applicable in many fields and areas. However, these theories have restrictions due to their parametrization tool on some specific parameters. On the other hand, when we compare our suggested structure to the hypersoft form of fuzzy sets, such as fuzzy hypersoft set, intuitionistic fuzzy hypersoft set [24], and Pythagorean fuzzy hypersoft set [28], we find that our proposed structure is superior.

The fuzzy hypersoft set, the intuitionistic fuzzy hypersoft set, and the Pythagorean fuzzy hypersoft set are all special cases of the q-rung orthopair fuzzy hypersoft set, as can be shown from these structures. Because our proposed structure provide more information by comparing with existing research. This proposed structure can deal with uncertain data in the decision making process in a very simple way. Therefore, the structure of q rung orthopair fuzzy hypersoft sets is more practical from the existing fuzzy structures. In our proposed structure, we use multiple choices of attributes in order to solve the problems of everyday life. Our proposed structure addressed the uncertainties in a more specific way as compared to the existing structures.

6. Conclusion

To control different types of cryptocurrencies, avoid losses, and continue trading in the online market, an effective and proper analysis of the cryptocurrency market is necessary. The analysis of the cryptocurrency market revealed that security, decentralization, and demand are the most essential elements for Bitcoin investment intentions, followed by financial incentives with a minor difference. Finally, the ranking of subfactors is the high level of security, decentralized application, and increased demand in the cryptocurrency market. As a result, a number of academics and researchers began working on cryptocurrency. Many scholars are turning to fuzzy set theory and its hybrid structures to solve the difficulty of studying the Bitcoin market since ambiguity exists in almost all real-world systems. In this work, a novel scientific instrument is designed that uses a parametric method to expose factual information. The q-rung orthopair fuzzy set and the hypersoft set define the multi-argument functions that provide the set of the q-rung orthopair fuzzy hypersoft. Two aggregation techniques for the q-rung orthopair fuzzy hypersoft sets are weighted mean and weighted geometric. The validity and implementation of the suggested operations and definitions are tested using appropriate examples. This research is also important for decision-making approaches. The main motive of experts in this work is to invest in the cryptocurrency market using a decision-making technique for various attributes and subattributes. In the future, we intend to create new techniques for analyzing the Bitcoin market using decision-making problems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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