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Analytic description to the fuzzy efficiencies in fuzzy standard Data Envelopment Analysis

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Abstract

In this paper we show how a parametric discussion on the optimal objective values to two mathematical models involved in a fuzzy standard data envelopment analysis can provide an analytic description to the membership functions of the fuzzy efficiencies of the decision-making units. We recall the mathematical models under discussion from the literature, but we approach them from a novel perspective, thus providing an analytical alternative to the numerical methods used so far in the literature.

Keywords: Data envelopment analysis, fuzzy efficiency, parametric programming
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1. Introduction

Data Envelopment Analysis (DEA) in fuzzy environment is a research field of wide interest in the recent literature. A relevant reference book in this field is [5]. More recently, Soltanzadeh and Omrani [9] proposed a model with fuzzy inputs and outputs for a dynamic network data envelopment analysis and applied it to evaluating the Iranian airlines companies. Their approach was based on the extension principle, and involved α-cuts to derive the fuzzy efficiencies of the decision-making units (DMUs).

A new model to rank the decision-making units within a DEA analysis was proposed in [13]. This model was applied to instances whose dataset uncertainty was expressed by several alternative scenarios.

A two-stage DEA for the efficiency evaluation in a fuzzy environment was developed in [6]. They identified the sources of inefficiency in a production system; explored the options for improving its performance; and derived a common set of weights to assure a balance among the evaluation of the system efficiency and the component process efficiencies. Azadi et al. [1] developed an integrated non-radial DEA model, and introduced a new efficiency measure to measure sustainable supplier performance.

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Zhou and Xu [14] presented a survey on fuzzy DEA researches and their successful applications. They equally reviewed the existing methodologies and applications providing useful clues to the researchers for further investigations in the field. The need of fuzzy logic in describing uncertainty was re-emphasized in [12].

Returning to the roots of DEA, it is important to mention that Charnes et al. [4] proposed the classic / standard / CCR (Charnes-Cooper-Rhodes) DEA model. The model was extended by Banker et al. [2] who included variable returns to scale along with the original input and output variable weights. A fuzzy extension to their model was discussed by Kao and Liu [7]. They provided a numerical description of the membership functions of the fuzzy efficiencies of DMUs based on the α-cuts of the fuzzy parameters. In this study we recall the mathematical models proposed by Kao and Liu [7]; approach them from a novel perspective – a parametric analysis with respect to the membership degree α of the optimal objective value; and improve the numerical methods used so far in the literature.

The rest of the paper is organized as follows: the problem formulation is given in Section 2; the parametric analysis is described in Section 3; an illustrative example is provided in Section 4; and the final conclusion together with some directions for future research are included in Section 5.

2. Problem formulation

Given n decision-making units (DMUs) involved in an evaluation process, let x_{ij} denote the i-th input of the DMU$_j$, $i = 1, ..., m$, $j = 1, ..., n$; and let y_{rj} denote the r-th output for the DMU$_j$, $r = 1, ..., s$, $j = 1, ..., n$. In CCR DEA models (1), the efficiency of any DMU is equal to the maximum of the ratio of weighted outputs to weighted inputs subject to the constraint that for every DMU its corresponding ratio is less than or equal to unity.

Model (1) describes the efficiency θ_p^* of the DMU$_p$, $p = 1, ..., n$.

$$
\begin{aligned}
\theta_p^* = \max \quad & E_p(u, v) \\
\text{s.t.} \quad & \\
& E_j(u, v) \le 1, \quad j = \overline{1, n}, \\
& u, v \ge \varepsilon,
\end{aligned}
\tag{1}
$$

where $E_j(u, v) = \left(\sum_{r=1}^{s} u_r y_{rj} \right) / \left(\sum_{i=1}^{m} v_i x_{ij} \right)$, $j = 1, ..., n$. The weights of the inputs and outputs u and v respectively are the variables whose values have to be determined. ε is a small positive quantity (e.g. $\varepsilon = 10^{-7}$) which assures that all inputs and outputs are considered in the evaluation, even with a minor weight. Model (1) can be linearized using the well-known Charnes-Cooper transformation [3] on the objective function $E_p(u, v)$; and multiplying each inequality $E_j(u, v) \le 1$ ($j = \overline{1, n}$) by its strict positive denominator. The linearized model is

$$
\begin{aligned}
\theta_p^* = \max \quad & \sum_{r=1}^{s} u_r y_{rp} \\
\text{s.t.} \quad & \\
& \sum_{i=1}^{m} v_i x_{ip} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij}, \quad j = \overline{1, n}, \\
& u, v \ge \varepsilon,
\end{aligned}
\tag{2}
$$

Banker et al. [2] considered variable returns to scale, and modified the CCR DEA model to

$$
\begin{aligned}
\theta_p^* = \max \quad & \sum_{r=1}^{s} u_r y_{rp} \\
\text{s.t.} \quad & \\
& v_0 + \sum_{i=1}^{m} v_i x_{ip} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} \le v_0 + \sum_{i=1}^{m} v_i x_{ij}, \quad j = \overline{1,n}, \\
& u, v \ge \varepsilon,
\end{aligned}
\tag{3}
$$

where v_0 is a variable without restrictions on its sign.

Kao and Liu [7] addressed the fuzzy DEA model (4) considering fuzzy inputs and outputs in Banker et al.'s DEA model, and defined the fuzzy efficiencies $\widetilde{\theta}_p^*$, $p = 1, \ldots, n$ as follows.

$$
\begin{aligned}
\widetilde{\theta}_p^* = \max \quad & \sum_{r=1}^{s} u_r \widetilde{y}_{rp} \\
\text{s.t.} \quad & \\
& v_0 + \sum_{i=1}^{m} v_i \widetilde{x}_{ip} = 1 \\
& \sum_{r=1}^{s} u_r \widetilde{y}_{rj} \le v_0 + \sum_{i=1}^{m} v_i \widetilde{x}_{ij}, \quad j = \overline{1,n}, \\
& u, v \ge \varepsilon,
\end{aligned}
\tag{4}
$$

They numerically constructed the approximate shapes for the fuzzy efficiencies $\widetilde{\theta}_p^*$, $p = 1, \ldots, n$ in accordance to the extension principle. They commented on some special cases when analytical solutions can be derived, but concluded that for their numerical example such solutions were not obtainable. Consequently, they used fifty-one equidistant values of α from 0 to 1, and numerically derived the shapes of the fuzzy efficiencies.

In the next section, we show how to obtain analytical solutions to fuzzy standard DEA problems as those considered in [7]. For our numerical illustration we recall Kao and Liu's example [7], and provide exact solutions that match to the approximate solutions from the literature.

More information on the fuzzy numbers used to describe the fuzzy parameters can be found in [11] and [16].

3. Parametric analysis

Kao and Liu [7] used the α-cut intervals of the fuzzy numbers that described the inputs and outputs in a fuzzy DEA model to construct a family of crisp linear programming problems whose optimal values provided the approximate shapes of DMUs fuzzy efficiencies. According to [7], for each fixed value of α, the optimal values to Models (5) and (6) describe the right and left endpoints respectively of the α-cut interval of the fuzzy efficiency of DMU_p.

More details on α-cut intervals and fuzzy mathematical programming can be found in [15].

Model

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rp}\right)_{\alpha}^{U} \\
\text{s.t.} \quad & \\
& v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ip}\right)_{\alpha}^{L} = 1 \\
& \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rj}\right)_{\alpha}^{L} \le v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ij}\right)_{\alpha}^{U}, \quad j = \overline{1,n}, j \ne p \\
& \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rp}\right)_{\alpha}^{U} \le v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ip}\right)_{\alpha}^{L} \\
& u, v \ge \varepsilon,
\end{aligned} \tag{5}$$

is used to describe the right side, while model

$$\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rp}\right)_{\alpha}^{L} \\
\text{s.t.} \quad & \\
& v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ip}\right)_{\alpha}^{U} = 1 \\
& \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rj}\right)_{\alpha}^{U} \le v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ij}\right)_{\alpha}^{L}, \quad j = \overline{1,n}, j \ne p \\
& \sum_{r=1}^{s} u_r \left(\widetilde{y}_{rp}\right)_{\alpha}^{L} \le v_0 + \sum_{i=1}^{m} v_i \left(\widetilde{x}_{ip}\right)_{\alpha}^{U} \\
& u, v \ge \varepsilon,
\end{aligned} \tag{6}$$

describes the left side.

The general notations $\left(\widetilde{A}\right)_{\alpha}^{L}$ and $\left(\widetilde{A}\right)_{\alpha}^{U}$ are used for the lower and upper endpoints of the α-cut interval $\left[\widetilde{A}\right]_{\alpha}$ of the fuzzy number $\widetilde{A}$. For a trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4)$, the α-cut interval is described by

$$\left(\widetilde{A}\right)_{\alpha}^{L} = (a_2 - a_1)\,\alpha + a_1, \quad \left(\widetilde{A}\right)_{\alpha}^{U} = (a_3 - a_4)\,\alpha + a_4 \ . \tag{7}$$

Both models (5) and (6) are parametric linear programming problems with respect to α when Formulas (7) are used for the lower and upper bounds of the fuzzy inputs and outputs. Adding $n + 2$ slack variables to each of the models (5) and (6), and replacing the variable v_0 by a difference of two positive variables, we obtain a general parametric linear programming problem in its standard form

$$\begin{aligned}
\max \quad & c^T(\alpha)\,u \\
\text{s.t.} \quad & \\
& A(\alpha)\,u = b, \\
& x \ge 0,
\end{aligned} \tag{8}$$

that may equally describe both previous models.

A parametric analysis of Problem (8) can be done by employing the optimality criterion. Starting from a basic feasible solution $u^* = \left(u_B^*, u_N^*\right)$ to Problem (8), we can split the matrix A in a quadratic sub-matrix A_B corresponding to the basic variables, and the sub-matrix A_N of non-basic variables such that $A = (A_B, A_N)$, A_B has a non-zero determinant, $u_B^*(\alpha) = A_B^{-1}(\alpha)\,b$, and $u_N^*(\alpha) = 0$. With respect to the base B that contains the indexes of variables included in $u_B^*(\alpha)$, the initial system of constraints $A(\alpha)\,u = b$ can be rewritten $A_B(\alpha)\,u_B + A_N(\alpha)\,u_N = b$ that is further equivalent to

$$u_B = A_B^{-1}(\alpha)\,b - A_B^{-1}(\alpha)\,A_N(\alpha)\,u_N.$$

Table 1. Triangular fuzzy numbers (TFN) and their α-cuts representing the input and output for four DMUs (first exmple)

DMUs	TFN-inputs	inputs α-cut	TFN-outputs	outputs α-cut
A	$(11, 12, 12, 14)$	$[11+\alpha, 14-2\alpha]$	$(10, 10, 10, 10)$	$[10, 10]$
B	$(30, 30, 30, 30)$	$[30, 30]$	$(12, 13, 14, 16)$	$[12+\alpha, 16-2\alpha]$
C	$(40, 40, 40, 40)$	$[40, 40]$	$(11, 11, 11, 11)$	$[11, 11]$
D	$(45, 47, 52, 55)$	$[45+2\alpha, 55-3\alpha]$	$(12, 15, 19, 22)$	$[12+3\alpha, 22-3\alpha]$

The optimality criterion that makes u^* to be optimal to Problem (8) imposes

$$c_B^T(\alpha) A_B^{-1}(\alpha) A_N(\alpha) - c_N^T(\alpha) \geq 0, \quad A_B^{-1}(\alpha) b \geq 0. \tag{9}$$

Since the parameters of Problem (8) are linear with respect to α, the components of the matrices involved in the above inequalities are ratios of polynomials of α, a discussion on their sign can be carried out, and piece-wise descriptions of the optimal values can be derived. A numerical illustration to these statements is given in the next section.

4. Numerical illustration

Our example is recalled from [7]. The same example was also considered in [10] and [8]. The fuzzy inputs and outputs used in Model (4) for this example are given in Table 1 as trapezoidal fuzzy numbers together with their corresponding α-cuts.

In what follows we carry out the parametric analysis needed to derive analytically the shape of the fuzzy efficiency of DMU D.

Model (10)

$$\begin{array}{ll} \max & (12+3\alpha)\, u_1 \\ \text{s.t.} & \\ & 10u_1 - v_0^+ + v_0^- - (11+\alpha)\, v_1 + s_1 = 0, \\ & (16-2\alpha)\, u_1 - v_0^+ + v_0^- - 30v_1 + s_2 = 0, \\ & 11u_1 - v_0^+ + v_0^- - 40v_1 + s_3 = 0, \\ & (12+3\alpha)\, u_1 - v_0^+ + v_0^- - (55-3\alpha)\, v_1 + s_4 = 0, \\ & -v_0^+ + v_0^- + (55-3\alpha)\, v_1 = 1, \\ & -u_1 + s_5 = \varepsilon, \\ & -v_1 + s_6 = \varepsilon \\ & u_1, v_0^+, v_0^-, v_1, s_1, s_2, s_3, s_4, s_5, s_6 \geq 0 \end{array} \tag{10}$$

is analyzed to obtain the increasing piece of the membership function of the fuzzy efficiency; while Model (11)

$$\begin{array}{ll} \max & (22-3\alpha)\, u_1 \\ \text{s.t.} & \\ & 10u_1 - v_0^+ + v_0^- - (14-2\alpha)\, v_1 + s_1 = 0, \\ & (12+\alpha)\, u_1 - v_0^+ + v_0^- - 30v_1 + s_2 = 0, \\ & 11u_1 - v_0^+ + v_0^- - 40v_1 + s_3 = 0, \\ & (22-3\alpha)\, u_1 - v_0^+ + v_0^- - (45+2\alpha)\, v_1 \left(\widetilde{x}_{14}\right)_\alpha^U + s_4 = 0, \\ & -v_0^+ + v_0^- + (45+2\alpha)\, v_1 = 1, \\ & -u_1 + s_5 = \varepsilon, \\ & -v_1 + s_6 = \varepsilon, \\ & u_1, v_0^+, v_0^-, v_1, s_1, s_2, s_3, s_4, s_5, s_6 \geq 0 \end{array} \tag{11}$$

is analyzed for deriving the decreasing piece of the same membership function.

Following [7], we use in Model (10) the lower bounds for the output of DMU D and the inputs of DMUs A, B, C; and upper bounds for the input of DMU D and the outputs of DMUs A, B, C, as prescribed in [7].

For $\alpha = 0$ in Model (10) the first optimal basis B_1 is formed with variables $\left\{u_1, v_0^+, v_1, s_1, s_3, s_4, s_5\right\}$. To discuss the optimality of this basis with respect to α we make use of the condition (9). Table 2 reports the values

Table 2. The Simplex Tableau that describes the optimality of bases B_1 and B_2 with respect to α when solving Model (10).

B_1	$A_{B_1}^{-1}(\alpha)\, b$	$A_{B_1}^{-1}(\alpha)\, A_N(\alpha)$		
u_1	$\frac{0.5}{8-\alpha}$	0	$\frac{0.5}{8-\alpha}$	$\frac{12.5-1.5\alpha}{8-\alpha}$
v_0^+	1	−1	0	$55-3\alpha$
v_1	0	0	0	−1
s_1	$\frac{3-\alpha}{8-\alpha}$	0	$\frac{-5}{8-\alpha}$	$\frac{203-34\alpha+\alpha^2}{8-\alpha}$
s_3	$\frac{2.5-\alpha}{8-\alpha}$	0	$\frac{-5.5}{8-\alpha}$	$\frac{-17.5-22.5\alpha+3\alpha^2}{8-\alpha}$
s_4	$\frac{2-2.5\alpha}{8-\alpha}$	0	$\frac{-6-1.5\alpha}{8-\alpha}$	$\frac{-150-19\alpha+4.5\alpha^2}{8-\alpha}$
s_5	$\frac{0.5}{8-\alpha}$	0	$\frac{0.5}{8-\alpha}$	$\frac{12.5-1.5\alpha}{8-\alpha}$
	$\frac{6+1.5\alpha}{8-\alpha}$	0	$\frac{6+1.5\alpha}{8-\alpha}$	$\frac{150+19\alpha-4.5\alpha^2}{8-\alpha}$

B_2	$A_{B_2}^{-1}(\alpha)\, b$			
u_1	$\frac{0.333}{4+\alpha}$	0	$\frac{0.333}{4+\alpha}$	0
v_0^+	1	−1	0	$55-3\alpha$
v_1	0	0	0	−1
s_1	$\frac{0.666+\alpha}{4+\alpha}$	0	$\frac{-3.333}{4+\alpha}$	$44-4\alpha$
s_3	$\frac{-1.333+1.666\alpha}{4+\alpha}$	0	$\frac{-5.333+0.667\alpha}{4+\alpha}$	$25-3\alpha$
s_2	$\frac{0.333+\alpha}{4+\alpha}$	0	$\frac{-3.667}{4+\alpha}$	$15-3\alpha$
s_5	$\frac{0.333}{4+\alpha}$	0	$\frac{0.333}{4+\alpha}$	0
	1	0	1	0

(depending on α, and with coefficients rounded to three decimal places) of the basic variables, and the quantities involved in the optimality criterion. For $\alpha \in [0,1]$ all components on the last row are positive as needed for optimality; and all values of the basic variables are positive except for s_4 that is positive if and only if $\alpha \leq 0.8$. It means that for any $\alpha \in [0, 0.8]$ the optimal value of Problem (10) is

$$\theta_4^*(\alpha) = \frac{6+1.5\alpha}{8-\alpha}. \tag{12}$$

Moreover, $\theta_4^*(0) = 0.75$ and $\theta_4^*(0.8) = 1$.

For $\alpha \geq 0.8$ the basic variable s_4 is replaced by s_2, and the basis $B_2 = \left\{u_1, v_0^+, v_1, s_1, s_3, s_2, s_5\right\}$ is obtained. Table 2 shows that B_2 is optimal for $\alpha \geq 0.8$ and the efficiency is $\theta_4^*(\alpha) = 1$.

We now proceed to analyse Model (11). This model uses the upper bounds for the output of DMU D and the inputs of DMUs A, B, C; and lower bounds for the input of DMU D and the outputs of DMUs A, B, C. Solving Model (11) firstly for $\alpha = 0$, and then for $\alpha \in (0,1]$, we obtain an optimal value equal to 1 for all values of $\alpha \in [0,1]$. Consequently, by inverting (12), and summarizing, we derive the following membership function of the fuzzy efficiency of DMU D

$$\mu_{\theta_D^*}(z) = \begin{cases} \dfrac{8z-6}{1.5+z}, & z \in [0.75, 1), \\ 1, & z = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

If we keep the coefficient of α^2 accurate (i.e. non-rounded to 0) in the expression of the optimal value in the Simplex Tableau that describes the basis B_1, namely if we work with $\frac{6+1.5\alpha+4\cdot 10^{-7}\alpha^2}{8-\alpha}$ instead of $\frac{6+1.5\alpha}{8-\alpha}$ seen on last row and second column in Table 2, then the membership function of the DMU D has the more complex form

$$\mu_{\theta_D^*}(z) = \begin{cases} \dfrac{2(8z-6)}{1.5+z+\sqrt{(1.5+z)^2+16(8z-6)10^{-7}}}, & z \in [0.75, 1), \\ 1, & z = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

However, it is clear that both membership functions (13) and (14) have similar shapes, and the second one can be reduced to the first one by rounding 10^{-7} to 0, in this latest step.

Table 3 summarizes the parametric results that yield the membership functions of the fuzzy efficiencies of all DMUs. Inverting the expressions given in Table 3, we derive the corresponding membership functions as follows:

$$\mu_{\theta_A^*}(z) = \begin{cases} 1, & z = 1, \\ 0, & \text{otherwise,} \end{cases}$$

$$\mu_{\theta_B^*}(z) = \begin{cases} \dfrac{15.3333+19.66667z-\sqrt{53.794889+1399.5291z-370.55529z^2}}{2(z-0.3333)}, & z \in [0.7183, 0.88867), \\ 1, & z \in [0.88867, 1], \\ 0, & \text{otherwise,} \end{cases}$$

Table 3. Optimal bases and optimal objective values that describe the membership functions of the fuzzy efficiencies of all DMUs

DMU	Lower optimal objective value		Upper optimal objective value	
A	$1,$	$\alpha \in [0,1]$	$1,$	$\alpha \in [0,1]$
B	$\frac{136+15.333\alpha+0.333\alpha^2}{189.333-19.667\alpha+\alpha^2},$	$\alpha \in [0,1]$	$1,$	$\alpha \in [0,1]$
C	$\frac{124.667+3.667\alpha}{229.333-29.667\alpha+\alpha^2},$	$\alpha \in [0,1]$	$\frac{11}{12+\alpha},$	$\alpha \in [0,0.000015]$
			$\frac{91.667-11\alpha}{100+3\alpha-\alpha^2},$	$\alpha \in (0.000015,1]$
D	$\frac{6+1.5\alpha+4\cdot 10^{-7}\alpha^2}{8-\alpha},$	$\alpha \in [0,0.8000065]$	$1,$	$\alpha \in [0,1]$
	$1,$	$\alpha \in (0.8000065,1]$		

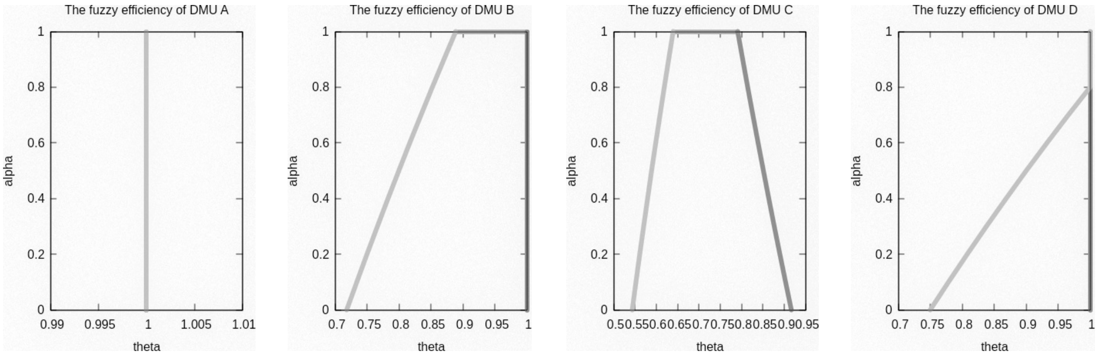

Fig. 1. Graphic representations of the fuzzy efficiencies

$$\mu_{\theta_C^*}(z) = \begin{cases} \dfrac{29.6667z + 3.6667 - \sqrt{13.444689 + 716.22436z - 37.221891z^2}}{2z}, & z \in [0.54366, 0.6935), \\ 1, & z = [0.6935, 0.7908], \\ \dfrac{11 + 3z + \sqrt{121 + 466z - 357.6668z^2}}{2z} & z \in (0.7908, 0.9166646], \\ \dfrac{10.9999 - 12z}{z} & z \in (0.9166646, 0.9166658], \\ 0, & \text{otherwise.} \end{cases}$$

Figure 1 shows the graphic representations of the membership functions of all DMUs involved in the evaluation process described in our numerical example. The obtained membership functions match the results reported in the literature.

5. Final remarks

In this paper we approached the fuzzy standard DEA through a novel parametric analysis of the optimal objective values of two crisp mathematical models. We proposed a methodology to provide analytic descriptions to the membership functions of the fuzzy efficiencies of DMUs. We founded our approach on the α-cuts of the fuzzy parameters, and a family of mathematical models recalled from the literature; advanced a new perspective; and succeeded to provide an analytical alternative to the numerical methods used so far in the literature. We also included a numerical illustration to our new solution approach.

In our future research we will focus on including a similar parametric analysis to other, more complex fuzzy DEA models, aiming to provide more accurate descriptions to the fuzzy efficiencies of DMUs involved in more sophisticated evaluation processes.

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