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Interval-valued intuitionistic fuzzy symmetric point criterion-based MULTIMOORA method for sustainable recycling partner selection in SMEs

Pratibha Rani[1] · Arunodaya Raj Mishra[2] · Dragan Pamucar[3] · Jabir Ali[4] · Ibrahim M. Hezam[5]

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Abstract

The need and strategy of eco-economy encourage enterprises to reach sustainability by employing sustainable supply chain management. Contrary to the numerous literatures focusing on green design and sustainability practices, this paper presents sustainable recycling partner (SRP) assessment with economic, environmental and social pillars. To propose an integrated framework for SRP selection in small-and-medium enterprises (SMEs), interval-valued intuitionistic fuzzy set (IVIFS)-based model is applied to deal with the vague, uncertain and qualitative information. Inspired by these topics, we propose IVIF-improved Dombi weighted averaging and IVIF-improved Dombi weighted geometric operators to aggregate the decision-making expert's preferences and discuss some sophisticated characteristics of developed aggregation operators. Further, we establish an integrated weighting model by combining the IVIF-symmetric point of criterion (IVIF-SPC) and IVIF-rank sum (IVIF-RS) tools. Then, the classical multi-objective optimization on the basis of ratio analysis plus full multiplicative form (MULTIMOORA) model has been extended using the proposed divergence measure and improved Dombi operators for treating multi-criteria decision analysis problems on IVIFS setting. To explore the effectiveness and practicability of the proposed model, a case study of SRP selection in SMEs is conducted. The results of the developed model, "Namo e-waste management limited (NEWML)," should be considered as the first SRP in SMEs in India. Further, the sensitivity investigation and comparative discussion are presented to check the stability and robustness of the presented technique.

Keywords Interval-valued intuitionistic fuzzy set · Divergence measure · Dombi operator · Symmetric point criterion · Rank sum · MULTIMOORA

1 Introduction

Sustainable supply chain management (SSCM) involves economic objectives, social performance and environmental considerations into the management of supply chains (Zhou et al. 2018). Due to increasing awareness of customers and stakeholders toward environmental issues, the business companies have started realizing the significance of technology by viewing it as an enabler of sustainability. Promoting sustainable and green practices help the firms in reducing their total carbon footprint as well as optimizing their end-to-end operations to accomplish effective cost savings and profitability (Zhu et al. 2023). Recycling and the material recovery-associated concern of sustainable development are achieving significance around the world due to its social, environmental and economic profits. The recycling practices for the end-of-life (EOL) materials and products conserve natural resources and energy, lessen greenhouse gases emissions and reduce air and water pollution. To perform the recycling businesses in small-and-medium enterprises (SMEs), it is required to select a suitable recycling partner which considers the economic, environmental and social aspects of sustainability in supply chain management (SCM) (Zhou et al. 2018). Due to involvement of multiple criteria and vague information, the sustainable recycling partner (SRP) selection procedure can be considered as a complex multi-criteria decision analysis (MCDA) problem (Rani et al. 2020; Więckowski et al. 2023). Few authors have developed some MCDA methods for handling the SRP selection

Extended author information available on the last page of the article

problems from different perspectives (Li et al. 2020; Mishra and Rani 2021; Badi et al. 2022). However, there is no study that deals with the SRP selection problem in which the performance values of partners concerning the criteria are given in terms of intervals rather than numerical value.

Interval-valued intuitionistic fuzzy set (IVIFS) (Atanassov and Gargov 1989) is a prevailing tool to deal with vagueness by considering both degree of membership and non-membership in an interval. The theory of IVIFS has widely been developed for treating uncertain information of realistic MCDA problems (Hajek and Froelich 2019; Das and Granados 2022; Gergin et al. 2022). Numerous scholars have employed IVIFSs to create MCDA tools for handling realistic issues (Garg and Kumar 2020). For instance, Xu (2007) discussed diverse aggregation operators (AOs) on IVIFSs to treat MCDA issues using the score and accuracy values for IVIFNs. Bai (2013) gave a modified score value to effectively prioritize the interval-valued intuitionistic fuzzy numbers (IVIFNs) and presented the decision support system to assess MCDA issue with fully unknown weighting data (Senapati et al. 2022; Ashraf et al. 2022; Zhou et al. 2022). Liu (2014) studied a set of Hamacher AOs and their application in group decision making. He presented several desirable characteristics of the proposed AOs. Wang and Mendel (2019) analyzed the drawbacks of existing AOs under IVIFS context. Moreover, they proposed an IVIF prioritized weighted averaging AO for MCDA with ordering of criteria. Jamil et al. (2020) established some induced generalized IVIF Einstein geometric AOs and their application in group MCDA from IVIF perspective. Further, Senapati et al. (2022) proposed some new AOs for IVIFSs and applied for solving MCDA problem with uncertain information.

Yeni and Özçelik (2019) suggested a hybrid combinative distance based assessment (CODAS) tool based on IVIFSs for treating the MCDA problem. Abdullah et al. (2019) discussed a model using Choquet integral and modified DEMATEL model with IVIFSs for treating sustainable solid waste assessment problems. Rani and Jain (2020) discussed some IVIF-information measures and compared with some extant measures. In addition, they proposed an algorithm for solving MCDA problems. Deveci et al. (2020) developed the CODAS tool with IVIF Euclidean and Taxicab distance to determine the renewable energy alternatives assessment problem using IVIFSs. Further, Mishra et al. (2020b) presented the TODIM tool with IVIF-information measures to choose suitable vehicle insurance firm on IVIFSs background. Alrasheedi et al. (2021) used the CoCoSo model on IVIFSs to prioritize and assess the factors of green growth to sustainable manufacturing in Malaysian firms. Wang et al. (2021) proposed a meta-evaluation model with best worst method (BWM) and multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) to solve the MCDA problems under IVIF environment.

The Dombi operator, introduced by Dombi (1982), is unusual since it has high parameter flexibility and can provide the parameter's sign whether it is conjunctive or disjunctive. He (2018) used the Dombi operations in a tentative, hazy environment while assessing the typhoon disasters. In a Pythagorean fuzzy environment, Akram et al. (2019) gave a set of Dombi AOs with important properties as idempotency, monotonicity, boundedness, reducibility and commutativity. Ashraf et al. (2020) developed numerous Dombi AOs under the framework of "spherical fuzzy set (SFS)," including the geometric, hybrid, Dombi weighted averaging, and discussed their features in detail. Kurama (2021) used some Dombi AOs through aggregation of similarities using the classifier. Karaaslan and Dawood (2021) proposed a series of Dombi weighted AOs for T-spherical fuzzy set. Saha et al. (2021) developed a set AOs on HFSs by combining the advantages of Archimedean and Dombi AOs and utilized these AOs to deal with personnel selection problem under HFSs. Liu et al. (2021) presented some general as well as flexible AOs combining the benefits of Dombi and Archimedean AOs to solve MCDA problems with the HFSs settings. Hu et al. (2020) discussed the duplex rating sets (DRSs) to express uncertain estimates and the associated confidence levels and gave transformation equations to convert DRSs into IVIFSs. They developed an entropy-weighted TOPSIS approach with IVIFSs information and applied to northwest China for supporting the assessment of clean energy-driven desalination-irrigation (CEDI) technology portfolios. Verma and Merigo (2020) discussed a flexible tool for IVIF MCDA problems with cosine similarity measure and presented elegant properties and proposed the weighted and ordered weighted IVIF cosine similarity measure. Alimoohammadlou and Khoshsepehr (2022) gave an ample MCDA model to assess sustainable development at companies. They proposed integrated with analytical hierarchy process (AHP) and the WASPAS on IVIFSs setting. Saha et al. (2022) discussed the MARCOS using generalized Dombi operator under dual probabilistic linguistic setting to deal with biomass feedstock selection problem. Kavitha et al. (2022) utilized the hesitant q-rung orthopair fuzzy Dombi AOs for feature selection. Unfortunately, there is no work regarding the improved Dombi AOs for IVIFSs.

In fuzzy set theory and its extensions, the notion of divergence measure (DM) is utilized to enumerate the discrimination between two sets. Due to its advantages, the development of different DMs and their applications in different areas is always a hot topic amongst the researchers (Zhang et al. 2010; Ye 2011; Mishra and Rani

2018; Bozanic et al. 2021; Khan et al. 2023). Kadian and Kumar (2020) discussed Renyi's-Tsallis fuzzy DM and applied for pattern recognition and fault detection. Rani and Jain (2020) introduced a new IVIF-DM to quantify the degree of discrimination between IVIFSs and presented its application. Verma (2021) investigated the shortcomings of existing intuitionistic fuzzy DMs. To conquer that issue, he introduced some order-α DMs and their several elegant properties. Khan et al. (2022) studied several DMs and their properties under circular intuitionistic fuzzy sets. In the context of IVIFSs, Mishra et al. (2020a, b) analyzed the drawbacks of existing measures and suggested some new IVIF-DMs (IVIF-DMs) with their applications in criteria weights' determination during service quality assessment and multi-criteria programming languages evaluation, respectively. Utilizing the idea of Jenson-Shannon divergence, Wang et al. (2022) planned a new IVIF-DM, which can reflect the degree of difference between IVIFSs.

The conception of MCDA has formed a basis for more rational and systematic decision, especially in the situation wherein several criteria need to be considered. It enables to select the most suitable option among a set of options by evaluating them in terms of numerous quantitative and qualitative criteria. In realistic MCDM problems, the decision-making expert's (DME's) estimations on considered criteria are often imprecise and subjective. Thus, the assessment and determination of positive and negative characteristics of one option relative to others are a multifaceted task for the DMEs. The MCDA techniques handle the process of making decisions in the occurrence of multiple criteria. One of the effective and flexible methods, named as multi-objective optimization on the basis of ratio analysis plus full multiplicative form (MULTIMOORA) (Brauers and Zavadskas 2010), combines the additive and multiplicative utility functions together with reference point approach, which makes it one of the robust MCDA approaches. The MULTIMOORA has great applicability in different environments such asanalyzing the barriers in renewable energy adoption (Asante et al. 2020), technology selection in nuclear power (Fernández and Vergara 2021), recognizing and observing the significant features affecting the slow adoption rate of blockchain technology (Siddiqui and Haroon 2023) and so forth.

The MOORA (Multi-objective optimization by ratio analysis) model (Brauers and Zavadskas 2006) is one of the renowned MCDA tools, which comprise two aggregation structures called the ratio system model (RSM) and the reference point system model (RPM). The MOORA model was further enhanced to the MULTIMOORA method by supplementing the full multiplicative form with the RSM and the RPM (Bari and Karande 2022). Comparing with the general MCDA tools like AHP, TOPSIS, VIKOR, ELECTRE, Brauers and Zavadskas (2012) demonstrated that the MULTIMOORA model has good stability, simple mathematical expressions, less computation time, and strong robustness (see Table 1). In the context of uncertainty, Zavadskas et al. (2015) established a hybrid IVIF information-based MULTIMOORA method without considering the DMEs and criteria weights. The classical MULTIMOORA has been extended into intuitionistic fuzzy environment (Zhang et al. 2019). They utilized their method for solving multi-criteria energy storage technologies assessment problem. Sarabi and Darestani (2021) developed a decision support system based on the BWM and MULTIMOORA with fuzzy information. In addition, they used their system to evaluate and rank the logistics service providers. An integrated MULTIMOORA has been presented using the maximum deviation model and Fermatean fuzzy set to solve the MCDM problems (Rani and Mishra 2021). Wang et al. (2021) presented an IVIF-BWM-MULTIMOORA model to analyze and determine the optimal alternative for a real case study. Liang et al. (2022) developed a novel generalization of MULTIMOORA method using triangular fuzzy numbers. The superiority and feasibility of their model have been presented through an application in hospital health-care delivery quality assessment problem. Shang et al. (2022) presented a new extension of MULTIMOORA using Shannon entropy, BWM and fuzzy information and further applied to evaluate the sustainable suppliers. Garg and Rani (2022) planned an integrated framework using the MULTIMOORA approach and IFS to deal with solid waste management techniques evaluation. Their framework utilized the concepts of particle swarm optimization and aggregation operators with MULTIMOORA, which can reflect the uncertainties during the MCDM process. Yu et al. (2023) presented the failure mode and effects assessment technique by the MULTIMOORA with interval asymmetric rough cloud tool for risk priority order. They implemented their model for risk assessment of the floating production storage and offloading single-point mooring system.

In the following, we present the key challenges and motivations behind the proposed work:

- Some authors (Zhou et al. 2018; Rani et al. 2020; Li et al. 2020; Mishra and Rani 2021) have presented different MCDA approaches for solving SRP selection problem with multiple sustainability indicators and uncertainty. However, these approaches are unable to

Table 1 Comparative studies of MCDA approaches (Brauers and Zavadskas 2012; Luo et al. 2019)

MCDA	Computational time	Simplicity	Mathematical computation	Stability	Information type
AHP	Very high	Very critical	Maximum	Poor	Mixed
TOPSIS	Moderate	Moderately critical	Moderate	Medium	Quantitative
VIKOR	Less	Simple	Moderate	Medium	Quantitative
ELECTRE	High	Moderately critical	Moderate	Medium	Mixed
PROMETHEE	High	Moderately critical	Moderate	Medium	Mixed
MULTIMOORA	Very less	Very simple	Minimum	Good	Quantitative

capture the uncertainty of multi-criteria SRP evaluation problem when the information is given in terms of interval.

- In 2009, Dombi (2009) presented the concept of improved Dombi operators, which are extension of algebraic operators, Einstein operators, hamacher operators and classical Dombi operators. Unfortunately, there is no study for aggregating the interval-valued intuitionistic fuzzy information using improved Dombi operators.
- IVIF-DM plays an important role in the field of decision making, image processing, pattern recognition and so forth. However, existing IVIF-DMs (Zhang et al. 2010; Ye 2011; Mishra and Rani 2018; Rani and Jain 2020) have some counter-intuitive cases.
- In the literature, several extensions of classical MULTIMOORA approach have been introduced (Zavadskas et al. 2015; Zhang et al. 2019; Sarabi and Darestani 2021; Rani and Mishra 2021; Wang et al. 2021; Liang et al. 2022; Shang et al. 2022; Garg and Rani 2022; Verma et al. 2022; Yu et al. 2023). However, these studies neglect the combination of objective and subjective criteria weights with interval-valued intuitionistic fuzzy information.
- In the process of MCDA, the determination of criteria weights is a major concern to make a proper decision. However, there is no work which computes the integrated criteria weights based on objective and subjective weights for multi-criteria SRP selection problem under interval-valued intuitionistic fuzzy environment.

Motivated by the concept of IVIFS and MULTIMOORA, this study develops a MCDA model to rank and assess the SRP in SMEs with identifying the limitations of extant studies. Here, we present the notable research contributions of this paper as.

- In order to rank SRPs in SMEs, this paper presents a hybrid IVIF-MULTIMOORA model using proposed divergence measure, AOs and integrated criteria weighting tool.
- This paper proposes the IVIF-symmetric point of criterion (IVIF-SPC) and IVIF-rank sum (IVIF-RS) model-based weighting approach to derive the criteria weights within IVIFS context.
- To aggregate the IVIFNs, this paper presents a new improved Dombi aggregation operator is developed with their enviable axioms.
- To avoid the limitation of existing IVIF-divergence measures, a new divergence measure has been proposed with their enviable properties to quantify the degree of discrimination between IVIFNs.
- To show superiority and feasibility of introduced model, a case study of SRPs selection in SMEs is taken. Also, comparison and sensitivity assessment are discussed further to illustrate the robustness and stability of the obtained results.

The rest part is summarized as follows: In Sect. 2, we provide some key ideas about IVIFSs, Dombi operations, and DM. In Sect. 3, we first present Dombi operations on IVIFNs and further develop some IVIF Dombi AOs and their characteristics. In Sect. 4, we discuss an IVIF-DM with its properties. In Sect. 5, we develop an integrated IVIF-SPC- MULTIMOORA model to tackle with MCDA problems. In Sect. 6, a case study of selecting right SRP in SMEs is presented to prove the effectiveness of introduced model. In addition, we show a comparative discussion to validate the advantage of the introduced approach. Conclusions of study with needful research direction are discussed in Sect. 7.

2 Preliminaries

In the current section, we present some fundamental ideas related to IVIFSs, Dombi operations and DM.

Definition 2.1 Let $\Omega = \{c_1, c_2, \ldots, c_n\}$ be a finite universal set. Atanassov and Gargov (1989) defined the mathematical form of IVIFS V on Ω as $V = \{(c_i, b_V(c_i), n_V(c_i)) : c_i \in \Omega\}$, where $b_V, n_V : V \to [0, 1]$ hold $\sup(b_V(c_i)) + \sup(n_V(c_i)) \leq 1$. The intervals $b_V(c_i)$ and $n_V(c_i)$ indicate the membership grade (MG) and non-membership grade (NG) of a variable c_i in V, respectively.

In other words, if $b_V(c_i) = \left[b_V^-(c_i), b_V^+(c_i)\right] \subset [0, 1]$ and $n_V(c_i) = \left[n_V^-(c_i), n_V^+(c_i)\right] \subset [0, 1]$, then $V = \left\{\left(c_i, \left[b_V^-(c_i), b_V^+(c_i)\right], \left[n_V^-(c_i), n_V^+(c_i)\right]\right) : c_i \in \Omega\right\}$, where $0 \leq b_V^+(c_i) + n_V^+(c_i) \leq 1, b_V^-(c_i) \geq 0$ and $n_V^-(c_i) \geq 0$.

The interval $\pi_V(c_i) = \left[\pi_V^-(c_i), \pi_V^+(c_i)\right] = \left[1 - b_V^+(c_i) - n_V^+(c_i),\ 1 - b_V^-(c_i) - n_V^-(c_i)\right]$ signifies the indeterminacy grade (IG) of c_i to V. The pair $\left(\left[b_V^-(c_i), b_V^+(c_i)\right], \left[n_V^-(c_i), n_V^+(c_i)\right]\right)$ is termed as an IVIFN (Xu 2007). For easiness, an IVIFN is commonly depicted as $\omega = ([b^-, b^+], [n^-, n^+])$, where $[b^-, b^+] \subset [0, 1]$, $[n^-, n^+] \subset [0, 1]$ and $b^+ + n^+ \leq 1$.

Definition 2.2 (Xu 2007). Suppose $\omega_1 = \left(\left[b_1^-, b_1^+\right], \left[n_1^-, n_1^+\right]\right)$ and $\omega_2 = \left(\left[b_2^-, b_2^+\right], \left[n_2^-, n_2^+\right]\right)$ be the IVIFNs. Then, the operations on IVIFNs can be defined by:

(a) $\omega_1 \subseteq \omega_2$ if and only if $b_1^-(c_i) \leq b_2^-(c_i)$, $b_1^+(c_i) \leq b_2^+(c_i)$, $n_1^-(c_i) \geq n_2^-(c_i)$ and $n_1^+(c_i) \geq n_2^+(c_i)$, $\forall c_i \in \Omega$,

(b) $\omega_1 = \omega_2$ if and only if $\omega_1 \subseteq \omega_2$ and $\omega_1 \supseteq \omega_2$,

(c) $\omega_1^c = \left\{\left(c_i, \left[n_1^-(c_i), n_1^+(c_i)\right], \left[b_1^-(c_i), b_1^+(c_i)\right]\right) \mid c_i \in \Omega\right\}$,

(d)
$$\omega_1 \cup \omega_2 = \left\{\left(\begin{matrix} c_i, & \left[b_1^-(c_i) \vee b_2^-(c_i), b_1^+(c_i) \vee b_2^+(c_i)\right], \\ & \left[n_1^-(c_i) \wedge n_2^-(c_i), n_1^+(c_i) \wedge n_2^+(c_i)\right] \end{matrix}\right) | c_i \in \Omega\right\},$$

(e)
$$\omega_1 \cap \omega_2 = \left\{\left(\begin{matrix} c_i, & \left[b_1^-(c_i) \wedge b_2^-(c_i), b_1^+(c_i) \wedge b_2^+(c_i)\right], \\ & \left[n_1^-(c_i) \vee n_2^-(c_i), n_1^+(c_i) \vee n_2^+(c_i)\right] \end{matrix}\right) | c_i \in \Omega\right\}.$$

Definition 2.3 (Xu 2007). Consider $\omega = \langle [b^-, b^+], [n^-, n^+] \rangle$ be an IVIFN. Then,

$$\mathbb{S}(\omega) = \frac{1}{2}\left(\frac{1}{2}(b^- + b^+ - n^- - n^+) + 1\right) \tag{1}$$

and

$$\mathbb{H}(\omega) = \frac{1}{2}(b^- + b^+ + n^- + n^+), \tag{2}$$

are the score and accuracy values of ω, respectively.

Definition 2.4 (Montes et al. 2015). Let $U, V, W \in IVIFSs(\Omega)$. An IVIF-DM $D_v : IVIFSs(\Omega) \times IVIFSs(\Omega) \to \mathbb{R}$ is a real-valued mapping which holds the following axioms:

(A1) $D_v(U, V) = D_v(V, U)$;
(A2) $D_v(U, V) = 0$ if and only if $U = V$;
(A3) $D_v(U \cap W, V \cap W) \leq D_v(U, V)$;
(A4) $D_v(U \cup W, V \cup W) \leq D_v(U, V)$.

Definition 2.5 (Dombi 1982). For any real numbers a_1 and a_2, the improved Dombi operator, denoted by $IDom_p^q$, is presented as:

$$\mathrm{IDom}_p^q(a_1, a_2) = \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \mho_p^q(a_k) - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \tag{3}$$

or

$$\mathrm{IDom}_p^q(a_1, a_2) = \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \Omega_p^q(a_k) - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \tag{4}$$

where $\Omega_p^q(a_k) = 1 + p\left(\frac{a_k}{1-a_k}\right)^q$, $\mho_p^q(a_k) = 1 + p\left(\frac{1-a_k}{a_k}\right)^q$, $a_k \in (0, 1)$, $k = 1, 2$ with $p > 0$ and $q \geq 1$.

Improved Dombi operations have decent superiority of variation over the parameters 'p' and 'q' which brand them advantageous compare to algebraic, Einstein and Hamacher operators.

3 Some improved Dombi AOs for IVIFSs

The current section firstly defines the Dombi operations on IVIFNs. Then, some improved Dombi AOs are proposed on IVIFNs with their enviable characteristics.

3.1 Improved Dombi operations on IVIFNs

Definition 3.1 For any two IVIFNs $\omega_1 = \left(\left[b_1^-, b_1^+\right], \left[n_1^-, n_1^+\right]\right)$ and $\omega_2 = \left(\left[b_2^-, b_2^+\right], \left[n_2^-, n_2^+\right]\right)$, the improved Dombi operations are defined on IVPFNs with $p > 0$ and $q \geq 1$:

$$\omega_1 \oplus \omega_2 = \left(\begin{array}{l} \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \Omega_p^q(b_k^-) - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \Omega_p^q(b_k^+) - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \mho_p^q(n_k^-) - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \mho_p^q(n_k^+) - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right] \end{array} \right); \tag{5}$$

$$\omega_1 \otimes \omega_2 = \left(\begin{array}{l} \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \mho_p^q(b_k^-) - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \mho_p^q(b_k^+) - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \Omega_p^q(n_k^-) - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{2} \Omega_p^q(n_k^+) - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right] \end{array} \right); \tag{6}$$

$$\zeta\,\omega_1 = \left(\begin{array}{l} \left[\left(1 + \left(\frac{1}{p}\left((\Omega_p^q(b_1^-))^{\zeta} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left((\Omega_p^q(b_1^+))^{\zeta} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left((\mho_p^q(n_1^-))^{\zeta} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left((\mho_p^q(n_1^+))^{\zeta} - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right] \quad (\zeta > 0); \end{array} \right) \tag{7}$$

$$\omega_k^{\zeta} = \left(\begin{array}{l} \left[\left(1 + \left(\frac{1}{p}\left((\mho_p^q(b_1^-))^{\zeta} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left((\mho_p^q(b_1^+))^{\zeta} - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left((\Omega_p^q(n_1^-))^{\zeta} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left((\Omega_p^q(n_1^+))^{\zeta} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right] \quad (\zeta > 0). \end{array} \right) \tag{8}$$

Theorem 3.1 *Let* $\omega_1 = \left(\left[b_1^-, b_1^+\right], \left[n_1^-, n_1^+\right]\right)$ *and* $\omega_2 = \left(\left[b_2^-, b_2^+\right], \left[n_2^-, n_2^+\right]\right)$ *be two IVIFNs and* $\zeta, \zeta_1, \zeta_2 > 0$. *Then, we have*:

(i) $\omega_1 \oplus \omega_2 = \omega_2 \oplus \omega_1$;
(ii) $\omega_1 \otimes \omega_2 = \omega_2 \otimes \omega_1$;
(iii) $\zeta(\omega_1 \oplus \omega_2) = (\zeta\,\omega_1) \oplus (\zeta\,\omega_2)$;
(iv) $\zeta(\omega_1 \otimes \omega_2) = (\zeta\,\omega_1) \otimes (\zeta\,\omega_2)$;
(v) $(\omega_1 \otimes \omega_2)^{\zeta} = (\omega_1^{\zeta}) \otimes (\omega_2^{\zeta})$;
(vi) $(\zeta_1 + \zeta_2)\,\omega_1 = (\zeta_1\omega_1) \oplus (\zeta_2\omega_1)$;
(vii) $(\omega_1)^{\zeta_1+\zeta_2} = (\omega_1^{\zeta_1}) \otimes (\omega_1^{\zeta_2})$.

3.2 IVIF-improved Dombi weighted averaging (IVIFIDWA) operator

Based on Definition 3.1, we propose IVIFIDWA operator and discuss their properties.

Definition 3.2 Let $\omega_k = \left\langle \left[b_k^-, b_k^+\right], \left[n_k^-, n_k^+\right] \right\rangle$ $(k = 1, 2, \ldots, s)$ be a collection of IVIFNs. Then, the IVIFIDWA operator is given by:

$$IVIFIDWA\,(\omega_1, \omega_2, \ldots, \omega_s) = (\lambda_1\,\omega_1) \oplus (\lambda_2\,\omega_2) \oplus (\lambda_3\,\omega_3) \oplus \ldots \oplus (\lambda_s\,\omega_s), \tag{9}$$

wherein $\lambda_k\,(k = 1(1)s)$ denotes the weight of $\omega_k\,(k = 1(1)s)$ with $\sum_{k=1}^{s} \lambda_k = 1$.

Theorem 3.2 *The aggregated value* $IVIFIDWA\,(\omega_1, \omega_2, \ldots, \omega_s)$ *is also an IVIFN. Moreover, we have.*

$$IVIFIDWA\ (\omega_1, \omega_2, ..., \omega_s) = \left(\begin{array}{c} \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\Omega_p^q(b_k^-))^{\lambda_k} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\Omega_p^q(b_k^+))^{\lambda_k} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\mho_p^q(n_k^-))^{\lambda_k} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\mho_p^q(n_k^+))^{\lambda_k} - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right] \end{array} \right). \tag{10}$$

Proof With the use of mathematical induction, we can easily prove this theorem.

Remark 3.1 Some particular cases of *IVIFIDWA* operator are presented as.

(i) If $p = 1$ and $q = 1$, then the *IVIFIDWA* reduces to the "IVIF weighted averaging" operator.
(ii) If $p = 1$ and $q = 2$, then the *IVIFIDWA* converts to the "IVIF Einstein weighted averaging" operator; and
(iii) If $q = 1$, then the *IVIFIDWA* operator reduces to the "IVIF hamacher weighted averaging" operator.

Next, the *IVIFIDWA* operator satisfies the following properties:

Property 3.1 (Shift invariance). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle\ (k = 1, 2, ..., s)$ be a collection of IVIFNs. Then, for an IVIFN $\omega_0 = \langle [b_0^-, b_0^+], [n_0^-, n_0^+] \rangle, (\neq \omega_k)$, we have:

$$IVIFIDWA\ (\omega_0 \oplus \omega_1, \omega_0 \oplus \omega_2, ..., \omega_0 \oplus \omega_s) = \omega_0 \oplus IVIFIDWA\ (\omega_1, \omega_2, ..., \omega_s).$$

Property 3.2 (Idempotency). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle\ (k = 1, 2, ..., s)$ be a collection of IVIFNs such that $\omega_k = \omega_0$(where $\omega_0 = \langle [b_0^-, b_0^+], [n_0^-, n_0^+] \rangle$), then $IVIFIDWA\ (\omega_1, \omega_2, ..., \omega_s) = \omega_0$.

Property 3.3 (Boundedness). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle\ (k = 1, 2, ..., s)$ be a collection of IVIFNs. Then $\omega^- \prec IVIFIDWA\ (\omega_1, \omega_2, ..., \omega_s) \prec \omega^+$, where $\omega^- = \langle [\min b_k^-, \min b_k^+], [\max n_k^-, \max n_k^+] \rangle$ and $\omega^+ = \langle [\max b_k^-, \max b_k^+], [\min n_k^-, \min n_k^+] \rangle$.

Property 3.4 (Monotonicity). Suppose $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle$ and $\omega_k' = \langle [b_k^{-\prime}, b_k^{+\prime}], [n_k^{-\prime}, n_k^{+\prime}] \rangle$ be the collections of IVIFNs, where $i = 1, 2, ..., s$. If $b_k^{-\prime} \le b_k^-, b_k^{+\prime} \le b_k^+, n_k^{-\prime} \ge n_k^-$ and $n_k^{+\prime} \ge n_k^+$, then $IVIFIDWA(\omega_1', \omega_2', ..., \omega_s') \le IVIFIDWA(\omega_1, \omega_2, ..., \omega_s)$.

3.3 IVIF-improved Dombi weighted geometric (IVIFIDWG) operator

Based on Definition 3.1, we introduce *IVIFIDWG* operator with its desirable properties.

Definition 3.3 Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle\ (k = 1, 2, ..., s)$ be a collection of IVIFNs. Then, the IVIFIDWG operator is given by.

$$IVIFIDWG(\omega_1, \omega_2, ..., \omega_s) = (\omega_1^{\lambda_1}) \otimes (\omega_2^{\lambda_2}) \otimes (\omega_3^{\lambda_3}) \otimes ... \otimes (\omega_s^{\lambda_s}), \tag{11}$$

where $\lambda_k\ (k = 1(1)s)$ denotes the weight of ω_k with $\sum_{k=1}^{s} \lambda_k = 1$.

Theorem 3.3 *The aggregated value IVIFIDWG* $(\omega_1, \omega_2, ..., \omega_s)$ *is also an IFN. Moreover, we have:*

$$IVIFIDWG\ (\omega_1, \omega_2, ..., \omega_s) = \left(\begin{array}{c} \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\mho_p^q(b_k^-))^{\lambda_k} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\mho_p^q(b_k^+))^{\lambda_k} - 1\right)\right)^{\frac{1}{q}}\right)^{-1} \right], \\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\Omega_p^q(n_k^-))^{\lambda_k} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{k=1}^{l} (\Omega_p^q(n_k^+))^{\lambda_k} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1} \right] \end{array} \right). \tag{12}$$

Proof It is similar as Theorem 3.2.

Remark 3.2 Some particular cases of *IVIFIDWG* operator are presented as.

(i) If $p = 1$ and $q = 1$, then the *IVIFIDWG* reduces to the "IVIF weighted geometric" operator.
(ii) If $p = 1$ and $q = 2$, then the *IVIFIDWA* converts to the "IVIF Einstein weighted geometric" operator; and
(iii) If $q = 1$, then the *IVIFIDWA* operator reduces to the "IVIF hamacher weighted geometric" operator.

Next, the *IVIFIDWG* operator satisfies the following properties:

Property 3.5 (Shift invariance). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle$ $(k = 1, 2, \ldots, s)$ be a collection of IVIFNs. Then, for an IVIFN $\omega_0 = \langle [b_0^-, b_0^+], [n_0^-, n_0^+] \rangle, (\neq \omega_k)$, we have:

$$IVIFIDWG\ (\omega_0 \oplus \omega_1, \omega_0 \oplus \omega_2, \ldots, \omega_0 \oplus \omega_s) = \omega_0 \oplus IVIFIDWG\ (\omega_1, \omega_2, \ldots, \omega_s).$$

Property 3.6 (Idempotency). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle$ $(k = 1, 2, \ldots, s)$ be a collection of IVIFNs such that $\omega_k = \omega_0$(where $\omega_0 = \langle [b_0^-, b_0^+], [n_0^-, n_0^+] \rangle$), then *IVIFIDWG* $(\omega_1, \omega_2, \ldots, \omega_s) = \omega_0$.

Property 3.7 (Boundedness). Let $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle$ $(k = 1, 2, \ldots, s)$ be a collection of IVIFNs. Then $\omega^- \prec IVIFIDWG\ (\omega_1, \omega_2, \ldots, \omega_s) \prec \omega^+$, where $\omega^- = \langle [\min b_k^-, \min b_k^+], [\max n_k^-, \max n_k^+] \rangle$ and $\omega^+ = \langle [\max b_k^-, \max b_k^+], [\min n_k^-, \min n_k^+] \rangle$.

Property 3.8 (Monotonicity). Suppose $\omega_k = \langle [b_k^-, b_k^+], [n_k^-, n_k^+] \rangle$ and $\omega_k' = \langle [b_k^{-\prime}, b_k^{+\prime}], [n_k^{-\prime}, n_k^{+\prime}] \rangle$ be the collections of IVIFNs, where $i = 1, 2, \ldots, s$. If $b_k^{-\prime} \leq b_k^-, b_k^{+\prime} \leq b_k^+, n_k^{-\prime} \geq n_k^-$ and $n_k^{+\prime} \geq n_k^+$, then

$$IVIFIDWG(\omega_1', \omega_2', \ldots, \omega_s') \leq IVIFIDWG(\omega_1, \omega_2, \ldots, \omega_s).$$

4 Proposed divergence measure for IVIFNs

In this section, we firstly present the drawbacks of exiting divergence measures under IVIFS environment for any two IVIFNs $\omega_1 = ([b_1^-, b_1^+], [n_1^-, n_1^+])$ and $\omega_2 = ([b_2^-, b_2^+], [n_2^-, n_2^+])$.

Zhang et al. (2010):

$$D_Z(\omega_1, \omega_2) = \sum_{i=1}^{n} \left[\mu_1(c_i) \ln\left(\frac{\mu_1(c_i)}{(\mu_1(c_i) + \mu_2(c_i))/2} \right) + (1 - \mu_1(c_i)) \ln\left(\frac{1 - \mu_1(c_i)}{(2 - \mu_1(c_i) - \mu_2(c_i))/2} \right) \right],$$

where

$$\mu_1(c_i) = \frac{(\mu_1^-(c_i) + \mu_1^+(c_i) + 2 - v_1^-(c_i) - v_1^+(c_i))}{4},$$

$$\mu_2(c_i) = \frac{(\mu_2^-(c_i) + \mu_2^+(c_i) + 2 - v_2^-(c_i) - v_2^+(c_i))}{4}.$$

Ye (2011):

$$D_Y(\omega_1, \omega_2) = \sum_{i=1}^{n} \left[\left(\frac{(\mu_1^-(c_i) + \mu_1^+(c_i) + 2 - v_1^-(c_i) - v_1^+(c_i))}{4} \right) \cdot \log_2 \frac{(\mu_1^-(c_i) + \mu_1^+(c_i) + 2 - v_1^-(c_i) - v_1^+(c_i))}{((\mu_1^-(c_i) + \mu_1^+(c_i) + 2 - v_1^-(c_i) - v_1^+(c_i)) + (\mu_2^-(c_i) + \mu_2^+(c_i) + 2 - v_2^-(c_i) - v_2^+(c_i)))/2} \right.$$
$$\left. \sum_{i=1}^{n} \left(\frac{(v_1^-(c_i) + v_1^+(c_i) + 2 - \mu_1^-(c_i) - \mu_1^+(c_i))}{4} \right) \cdot \log_2 \frac{(v_1^-(c_i) + v_1^+(c_i) + 2 - \mu_1^-(c_i) - \mu_1^+(c_i))}{((v_1^-(c_i) + v_1^+(c_i) + 2 - \mu_1^-(c_i) - \mu_1^+(c_i)) + (v_2^-(c_i) + v_2^+(c_i) + 2 - \mu_2^-(c_i) - \mu_2^+(c_i)))/2} \right].$$

Mishra and Rani (2018):

$$
\begin{aligned}
D_M(\omega_1,\,\omega_2) = & \frac{1}{n\left(e^{1/2}-1\right)}\sum_{i=1}^{n}\left[\left\{\frac{\begin{array}{c}\left(\mu_1^-(c_i)+\mu_2^-(c_i)\right)+\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\,+4\\ -\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,-\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)\end{array}}{8}\right.\right.\\
& \cdot\exp\left(\frac{\begin{array}{c}\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,+\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\,+4\\ -\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,-\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)\end{array}}{8}\right)+\frac{\begin{array}{c}\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,+\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)+4\\ -\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,-\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\end{array}}{8}\\
& \exp\left(\frac{\begin{array}{c}\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,+\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)+4\\ -\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,-\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\end{array}}{8}\right)-\frac{1}{2}\left\{\frac{\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)\,+2-\,\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)}{4}\right.\\
& \cdot\exp\left(\frac{\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)\,+2-\,\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)}{4}\right)+\frac{\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)\,+2-\,\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)\,+2-\,\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)}{4}\right)+\frac{\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)\,+2-\,\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)\,+2-\,\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)}{4}\right)+\frac{\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)\,+2-\,\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)\,+2-\,\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)}{4}\right).
\end{aligned}
$$

Rani and Jain (2020):

$$
\begin{aligned}
D_R(\omega_1,\,\omega_2) = & \frac{1}{n\left(e^{1/2}-1\right)}\sum_{i=1}^{n}\left[\left\{\frac{\begin{array}{c}\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,+\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\,+4\\ -\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,-\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)\end{array}}{8}\right.\right.\\
& \cdot\exp\left(\frac{\begin{array}{c}\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,+\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)+4\\ -\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,-\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\end{array}}{8}\right)+\frac{\begin{array}{c}\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,+\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)+4\\ -\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,-\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\end{array}}{8}\\
& \exp\left(\frac{\begin{array}{c}\left(\mu_1^-(c_i)\,+\,\mu_2^-(c_i)\right)\,+\,\left(\mu_1^+(c_i)\,+\,\mu_2^+(c_i)\right)\,+4\\ -\left(\nu_1^-(c_i)\,+\,\nu_2^-(c_i)\right)\,-\,\left(\nu_1^+(c_i)\,+\,\nu_2^+(c_i)\right)\end{array}}{8}\right)-\frac{1}{2}\left\{\frac{\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)\,+2-\,\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)}{4}\right.\\
& \cdot\exp\left(\frac{\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)\,+2-\,\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)}{4}\right)+\frac{\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)\,+2-\,\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\mu_1^-(c_i)\,+\,\mu_1^+(c_i)\right)\,+2-\,\left(\nu_1^-(c_i)\,+\,\nu_1^+(c_i)\right)}{4}\right)+\frac{\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)\,+2-\,\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)\,+2-\,\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)}{4}\right)+\frac{\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)\,+2-\,\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)}{4}\\
& \cdot\exp\left(\frac{\left(\mu_2^-(c_i)\,+\,\mu_2^+(c_i)\right)\,+2-\,\left(\nu_2^-(c_i)\,+\,\nu_2^+(c_i)\right)}{4}\right).
\end{aligned}
$$

The following examples show the drawbacks of aforesaid IVIF-divergence measures:

Example 4.1 For any two IVIFNs $\omega_1 = ([0.1, 0.1], [0.3, 0.3])$ and $\omega_2 = ([0.2, 0.2], [0.4, 0.4])$, the existing measures are $D_Z(\omega_1, \omega_2) = 0$, $D_Y(\omega_1, \omega_2) = 0$, $D_M(\omega_1, \omega_2) = 0$ and $D_R(\omega_1, \omega_2) = 0$, but $\omega_1 \neq \omega_2$. It means that these measures are not satisfying the axiom (A2) of Definition 2.4 for given IVIFNs.

Example 4.2 For any two IVIFNs $\omega_1 = ([0.2, 0.4], [0.2, 0.4])$ and $\omega_2 = ([0.3, 0.5], [0.3, 0.5])$, the existing measures are $D_Z(\omega_1, \omega_2) = 0$, $D_Y(\omega_1, \omega_2) = 0$, $D_M(\omega_1, \omega_2) = 0$ and $D_R(\omega_1, \omega_2) = 0$, but $\omega_1 \neq \omega_2$. Therefore, the measures given by (Zhang et al. 2010; Ye 2011; Mishra and Rani 2018; Rani and Jain 2020) do not satisfy the axiom (A2) of Definition 2.4 for given IVIFNs.

Definition 4.1 Suppose $\omega_1 = \left(\left[b_1^-, b_1^+\right], \left[n_1^-, n_1^+\right]\right)$ and $\omega_2 = \left(\left[b_2^-, b_2^+\right], \left[n_2^-, n_2^+\right]\right)$ be two IVIFNs. Now, the divergence measure for IVIFNs is presented as

$$\begin{aligned} D(\omega_1, \omega_2) = & \frac{1}{4n\,(\exp(2) - 1)} \sum_{i=1}^{n} \Bigg[\left(b_1^-(c_i) - b_2^-(c_i)\right) \left(\exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)} \right) - \exp\left(\frac{4\,b_2^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)} \right) \right) \\ & + \left(b_1^+(c_i) - b_2^+(c_i)\right) \left(\exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)} \right) - \exp\left(\frac{4\,b_2^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)} \right) \right) \\ & + \left(n_1^-(c_i) - n_2^-(c_i)\right) \left(\exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)} \right) - \exp\left(\frac{4\,n_2^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)} \right) \right) \\ & + \left(n_1^+(c_i) - n_2^+(c_i)\right) \left(\exp\left(\frac{4\,n_1^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)} \right) - \exp\left(\frac{4\,n_2^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)} \right) \right) \Bigg]. \end{aligned} \tag{13}$$

Theorem 4.1 *The function $D(\omega_1, \omega_2)$ given in Eq. (13) is a valid IVIF-DM.*

Proof To proof this theorem, the function $D(\omega_1, \omega_2)$ must have to fulfill the axioms of Definition 2.4.

(A1) It is obvious that $D(\omega_1, \omega_2) = D(\omega_2, \omega_1)$.

(A2) For $\omega_1, \omega_2 \in IVIFSs(\Omega)$, if $D(\omega_1, \omega_2) = 0$, then

$$\begin{aligned} & \left(b_1^-(c_i) - b_2^-(c_i)\right) \left(\exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)} \right) - \exp\left(\frac{4\,b_2^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)} \right) \right) \\ & + \left(b_1^+(c_i) - b_2^+(c_i)\right) \left(\exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)} \right) - \exp\left(\frac{4\,b_2^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)} \right) \right) \\ & + \left(n_1^-(c_i) - n_2^-(c_i)\right) \left(\exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)} \right) - \exp\left(\frac{4\,n_2^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)} \right) \right) \\ & + \left(n_1^+(c_i) - n_2^+(c_i)\right) \left(\exp\left(\frac{4\,n_1^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)} \right) - \exp\left(\frac{4\,n_2^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)} \right) \right) = 0. \end{aligned}$$

Since all four terms are negative for input values, therefore, we have

$$\left(b_1^-(c_i) - b_2^-(c_i)\right)\left(\exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right) - \exp\left(\frac{4\,b_2^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right)\right) = 0,$$

$$\left(b_1^+(c_i) - b_2^+(c_i)\right)\left(\exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right) - \exp\left(\frac{4\,b_2^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right)\right) = 0,$$

$$\left(n_1^-(c_i) - n_2^-(c_i)\right)\left(\exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right) - \exp\left(\frac{4\,n_2^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right)\right) = 0$$

and $\left(n_1^+(c_i) - n_2^+(c_i)\right)\left(\exp\left(\frac{4\,n_1^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right) - \exp\left(\frac{4\,n_2^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right)\right) = 0.$

This implies that:

$\left(b_1^-(c_i) - b_2^-(c_i)\right) = 0$ and $\exp\left(\frac{4\,b_1^-(c_i)}{1+b_1^-(c_i)+b_2^-(c_i)}\right) - \exp\left(\frac{4\,b_2^-(c_i)}{1+b_1^-(c_i)+b_2^-(c_i)}\right) = 0,$

$\left(b_1^+(c_i) - b_2^+(c_i)\right) = 0$ and $\exp\left(\frac{4\,b_1^+(c_i)}{1+b_1^+(c_i)+b_2^+(c_i)}\right) - \exp\left(\frac{4\,b_2^+(c_i)}{1+b_1^+(c_i)+b_2^+(c_i)}\right) = 0,$

$\left(n_1^-(c_i) - n_2^-(c_i)\right) = 0$ and $\exp\left(\frac{4\,n_1^-(c_i)}{1+n_1^-(c_i)+n_2^-(c_i)}\right) - \exp\left(\frac{4\,n_2^-(c_i)}{1+n_1^-(c_i)+n_2^-(c_i)}\right) = 0,$

$\left(n_1^+(c_i) - n_2^+(c_i)\right) = 0$ and $\exp\left(\frac{4\,n_1^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right) - \exp\left(\frac{4\,n_2^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right) = 0.$

From all these cases, we get $b_1^-(c_i) = b_2^-(c_i)$, $b_1^+(c_i) = b_2^+(c_i)$, $n_1^-(c_i) = n_2^-(c_i)$ and $n_1^+(c_i) = n_2^+(c_i)$. Thus, $\omega_1 = \omega_2$. Similarly, we can prove that if $\omega_1 = \omega_2$, then $D(\omega_1, \omega_2) = 0$.

(A3) $D(\omega_1 \cap \omega_3,\, \omega_2 \cap \omega_3) = \frac{1}{4\,n\,(\exp(2)-1)}$

$$\sum_{i=1}^{n}\left[\begin{aligned}&\begin{pmatrix}\min\{b_1^-(c_i), b_3^-(c_i)\}\\ -\min\{b_2^-(c_i), b_3^-(c_i)\}\end{pmatrix}\begin{pmatrix}\exp\left(\dfrac{4\min\{b_1^-(c_i), b_3^-(c_i)\}}{1+\min\{b_1^-(c_i), b_3^-(c_i)\}+\min\{b_2^-(c_i), b_3^-(c_i)\}}\right)\\ -\exp\left(\dfrac{4\min\{b_2^-(c_i), b_3^-(c_i)\}}{1+\min\{b_1^-(c_i), b_3^-(c_i)\}+\min\{b_2^-(c_i), b_3^-(c_i)\}}\right)\end{pmatrix}\\ &+\begin{pmatrix}\min\{b_1^+(c_i), b_3^+(c_i)\}-\\ \min\{b_2^+(c_i), b_3^+(c_i)\}\end{pmatrix}\begin{pmatrix}\exp\left(\dfrac{4\min\{b_1^+(c_i), b_3^+(c_i)\}}{1+\min\{b_1^+(c_i), b_3^+(c_i)\}+\min\{b_2^+(c_i), b_3^+(c_i)\}}\right)\\ -\exp\left(\dfrac{4\min\{b_2^+(c_i), b_3^+(c_i)\}}{1+\min\{b_1^+(c_i), b_3^+(c_i)\}+\min\{b_2^+(c_i), b_3^+(c_i)\}}\right)\end{pmatrix}\\ &+\begin{pmatrix}\max\{n_1^-(c_i), n_3^-(c_i)\}\\ -\max\{n_2^-(c_i), n_3^-(c_i)\}\end{pmatrix}\begin{pmatrix}\exp\left(\dfrac{4\max\{n_1^-(c_i), n_3^-(c_i)\}}{1+\max\{n_1^-(c_i), n_3^-(c_i)\}+\max\{n_2^-(c_i), n_3^-(c_i)\}}\right)\\ -\exp\left(\dfrac{4\max\{n_2^-(c_i), n_3^-(c_i)\}}{1+\max\{n_1^-(c_i), n_3^-(c_i)\}+\max\{n_2^-(c_i), n_3^-(c_i)\}}\right)\end{pmatrix}\\ &+\begin{pmatrix}\max\{n_1^+(c_i), n_3^+(c_i)\}\\ -\max\{n_2^+(c_i), n_3^+(c_i)\}\end{pmatrix}\begin{pmatrix}\exp\left(\dfrac{4\max\{n_1^+(c_i), n_3^+(c_i)\}}{1+\max\{n_1^+(c_i), n_3^+(c_i)\}+\max\{n_2^+(c_i), n_3^+(c_i)\}}\right)\\ -\exp\left(\dfrac{4\max\{n_2^-(c_i), n_3^-(c_i)\}}{1+\max\{n_1^+(c_i), n_3^+(c_i)\}+\max\{n_2^+(c_i), n_3^+(c_i)\}}\right)\end{pmatrix}\end{aligned}\right]. \tag{14}$$

From $\min\{b_1^-(c_i), b_3^-(c_i)\}$, $\min\{b_2^-(c_i), b_3^-(c_i)\}$, $\min\{b_1^+(c_i), b_3^+(c_i)\}$, $\min\{b_2^+(c_i), b_3^+(c_i)\}$, $\max\{n_1^-(c_i), n_3^-(c_i)\}$, $\max\{n_2^-(c_i), n_3^-(c_i)\}$, $\max\{n_1^+(c_i), n_3^+(c_i)\}$ and $\max\{n_2^+(c_i), n_3^+(c_i)\}$, we deduce the following results:

$$b_1^-(c_i) \le b_3^-(c_i) \le b_2^-(c_i) \text{ or } b_2^-(c_i) \le b_3^-(c_i) \le b_1^-(c_i),\ b_1^+(c_i) \le b_3^+(c_i) \le b_2^+(c_i) \text{ or } b_2^+(c_i) \le b_3^+(c_i) \le b_1^+(c_i), \tag{15}$$

$$b_3^-(c_i) \le \{b_1^-(c_i)\ \&\ b_2^-(c_i)\} \text{ or } b_3^-(c_i) \ge \{b_1^-(c_i)\ \&\ b_2^-(c_i)\}, b_3^+(c_i) \le \{b_1^+(c_i)\ \&\ b_2^+(c_i)\} \text{ or } b_3^+(c_i) \ge \{b_1^+(c_i)\ \&\ b_2^+(c_i)\}, \tag{16}$$

$$n_1^-(c_i) \le n_3^-(c_i) \le n_2^-(c_i) \text{ or } n_2^-(c_i) \le n_3^-(c_i) \le n_1^-(c_i), n_1^+(c_i) \le n_3^+(c_i) \le n_2^+(c_i) \text{ or } n_2^+(c_i) \le n_3^+(c_i) \le n_1^+(c_i) \tag{17}$$

$$n_3^-(c_i) \le \{n_1^-(c_i)\ \&\ n_2^-(c_i)\} \text{ or } n_3^-(c_i) \ge \{n_1^-(c_i)\ \&\ n_2^-(c_i)\},\ or\ n_3^+(c_i) \le \{n_1^+(c_i)\ \&\ n_2^+(c_i)\} \text{ or } n_3^+(c_i) \ge \{n_1^+(c_i)\ \&\ n_2^+(c_i)\}. \tag{18}$$

The proof is easy for the situations in (16) and (18). Now, we prove the results for (15) and (17); then, Eq. (14) becomes

From Eqs. (15) and (17), we have

$$b_3^-(c_i) - b_1^-(c_i) \le b_2^-(c_i) - b_1^-(c_i),$$

$$\exp\left(\frac{4\,b_3^-(c_i)}{1 + b_1^-(c_i) + b_3^-(c_i)}\right) - \exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_3^-(c_i)}\right) \le \exp\left(\frac{4\,b_2^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right) - \exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right),$$

$$b_3^+(c_i) - b_1^+(c_i) \le b_2^+(c_i) - b_1^+(c_i),$$

$$\exp\left(\frac{4\,b_3^+(c_i)}{1 + b_1^+(c_i) + b_3^+(c_i)}\right) - \exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_3^+(c_i)}\right) \le \exp\left(\frac{4\,b_2^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right) - \exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right),$$

$$n_3^-(c_i) - n_1^-(c_i) \le n_2^-(c_i) - n_1^-(c_i),$$

$$\exp\left(\frac{4\,n_3^-(c_i)}{1 + n_1^-(c_i) + n_3^-(c_i)}\right) - \exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_3^-(c_i)}\right) \le \exp\left(\frac{4\,n_2^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right) - \exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right),$$

$$n_3^+(c_i) - n_1^+(c_i) \le n_2^+(c_i) - n_1^+(c_i)$$

$$\begin{aligned} D(\omega_1 \cap \omega_3, \omega_2 \cap \omega_3) = & \frac{1}{4\,n\,(\exp(2) - 1)} \\ & \sum_{i=1}^{n} \Bigg[\left(b_3^-(c_i) - b_1^-(c_i)\right) \left(\exp\left(\frac{4\,b_3^-(c_i)}{1 + b_1^-(c_i) + b_3^-(c_i)}\right) - \exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_3^-(c_i)}\right)\right) \\ & + \left(b_3^+(c_i) - b_1^+(c_i)\right) \left(\exp\left(\frac{4\,b_3^+(c_i)}{1 + b_1^+(c_i) + b_3^+(c_i)}\right) - \exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_3^+(c_i)}\right)\right) \\ & + \left(n_2^-(c_i) - n_3^-(c_i)\right) \left(\exp\left(\frac{4\,n_2^-(c_i)}{1 + n_2^-(c_i) + n_3^-(c_i)}\right) - \exp\left(\frac{4\,n_3^-(c_i)}{1 + n_2^-(c_i) + n_3^-(c_i)}\right)\right) \\ & + \left(n_2^+(c_i) - n_3^+(c_i)\right) \left(\exp\left(\frac{4\,n_2^+(c_i)}{1 + n_2^+(c_i) + n_3^+(c_i)}\right) - \exp\left(\frac{4\,n_3^+(c_i)}{1 + n_2^+(c_i) + n_3^+(c_i)}\right)\right) \Bigg] \end{aligned}$$

and

$$\begin{aligned} D(\omega_1, \omega_2) = & \frac{1}{4\,n\,(\exp(2) - 1)} \\ & \sum_{i=1}^{n} \Bigg[\left(b_1^-(c_i) - b_2^-(c_i)\right) \left(\exp\left(\frac{4\,b_1^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right) - \exp\left(\frac{4\,b_2^-(c_i)}{1 + b_1^-(c_i) + b_2^-(c_i)}\right)\right) \\ & + \left(b_1^+(c_i) - b_2^+(c_i)\right) \left(\exp\left(\frac{4\,b_1^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right) - \exp\left(\frac{4\,b_2^+(c_i)}{1 + b_1^+(c_i) + b_2^+(c_i)}\right)\right) \\ & + \left(n_1^-(c_i) - n_2^-(c_i)\right) \left(\exp\left(\frac{4\,n_1^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right) - \exp\left(\frac{4\,n_2^-(c_i)}{1 + n_1^-(c_i) + n_2^-(c_i)}\right)\right) \\ & + \left(n_1^+(c_i) - n_2^+(c_i)\right) \left(\exp\left(\frac{4\,n_1^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)}\right) - \exp\left(\frac{4\,n_2^+(c_i)}{1 + n_1^+(c_i) + n_2^+(c_i)}\right)\right) \Bigg]. \end{aligned}$$

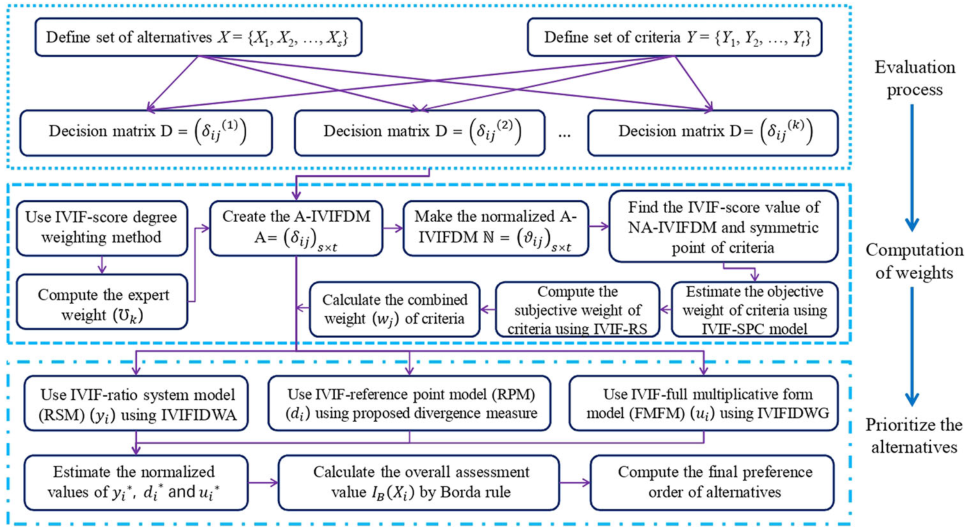

Fig. 1 Flowchart of the developed IVIF-SPC-RS-MULTIMOORA model

$$\exp\left(\frac{4\,n_3^+(c_i)}{1+n_1^+(c_i)+n_3^+(c_i)}\right)-\exp\left(\frac{4\,n_1^+(c_i)}{1+n_1^+(c_i)+n_3^+(c_i)}\right)$$
$$\leq \exp\left(\frac{4\,n_2^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right)-\exp\left(\frac{4\,n_1^+(c_i)}{1+n_1^+(c_i)+n_2^+(c_i)}\right).$$

This implies that $D(\omega_1 \cap \omega_3,\, \omega_2 \cap \omega_3) \leq D(\omega_1,\, \omega_2)$.

Thus, the measure Eq. (13) holds all the axioms of divergence measure on IVIFSs. Hence, it is a valid IVIF-DM.

Remark 4.1 For Example 4.1 and 4.2, the proposed divergence measure provides the value "0.0080" and "0.0083," respectively, while the measures proposed by (Zhang et al. 2010; Ye 2011; Mishra and Rani 2018; Rani and Jain 2020) are unable to discriminate the IVIFNs ω_1 and ω_2, given in Example 4.1 and 4.2.

5 An integrated IVIF-SPC-RS-MULTIMOORA method

This section introduces an MCDA model for solving decision-making problem using the SPC, RS and MULTIMOORA methods with IVIF information. The procedural steps are as follows (see Fig. 1):

Step 1 Construct an IVIF decision matrix (IVIFDM).

A team $G = \{g_1, g_2, \ldots, g_l\}$ of DMEs is created to evaluate a set of alternatives $X = \{X_1, X_2, \ldots, X_s\}$ with respect to criteria set $Y = \{Y_1, Y_2, \ldots, Y_t\}$. The created team presents his/her evaluation values for each option X_i over the criteria $Y_j\,(j = 1, 2, \ldots, t)$ in terms of linguistic values (LVs). Consider that $D = \left(\delta_{ij}^{(k)}\right)_{s\times t}$ be the linguistic decision matrix (LDM) offered by DMEs, where $\delta_{ij}^{(k)}$ denotes the LV of each option X_i over a criterion Y_j provided by kth DME. With the use of linguistic rating table, the LDM is converted into IVIFDM.

Step 2 Evaluate the weights of DMEs.

Let $\Gamma_k = \left(\left[b_k^-,\, b_k^+\right],\, \left[n_k^-,\, n_k^+\right]\right)$ be an IVIFN analogous to the LV allocated for the relative significance value of a DME g_k. Then, the numerical weight of kth DME is obtained as follows:

$$\mho_k = \frac{\left(b_k^- + b_k^+\right)\left(2+\pi_k^- + \pi_k^+\right)}{\sum_{k=1}^{\ell}\left(\left(b_k^- + b_k^+\right)\left(2+\pi_k^- + \pi_k^+\right)\right)},\ where\ \mho_k \in [0,\,1]\ and\ \sum_{k=1}^{l}\mho_k = 1. \tag{19}$$

Step 3 Create the aggregated IVIFDM (A-IVIFDM).

To combine the individual decision opinions, we create an A-IVIFDM $A = \left(\delta_{ij}\right)_{s\times t}$ with the help of IVIFIDWA operator (10) (or IVIFIDWG operator (12)), where

$$\delta_{ij} = \left(\left[b_{ij}^-, b_{ij}^+\right], \left[n_{ij}^-, n_{ij}^+\right]\right) = IVIFIDWA_{\mho}\left(\delta_{ij}^{(1)},\, \delta_{ij}^{(2)}, \ldots, \delta_{ij}^{(l)}\right) \tag{20}$$
$$or\ IVIFIDWG_{\mho}\left(\delta_{ij}^{(1)},\, \delta_{ij}^{(2)}, \ldots, \delta_{ij}^{(l)}\right).$$

Step 4 Generate the normalized A-IVIFDM (NA-IVIFDM).

In this MCDM procedure, the NA-IVIFDM $\mathbb{N} = \left(\vartheta_{ij}\right)_{s\times t}$ from A-IVIFDM $A = \left(\delta_{ij}\right)_{s\times t}$ is calculated, where

$$\vartheta_{ij} = \left(\left[\overline{b}_{ij}^-, \overline{b}_{ij}^+\right], \left[\overline{n}_{ij}^-, \overline{n}_{ij}^+\right]\right) = \begin{cases} \tilde{\delta}_{ij} = \left(\left[b_{ij}^-, b_{ij}^+\right], \left[n_{ij}^-, n_{ij}^+\right]\right), & j \in Y_b, \\ \left(\tilde{\delta}_{ij}\right)^c = \left(\left[n_{ij}^-, n_{ij}^+\right], \left[b_{ij}^-, b_{ij}^+\right]\right), & j \in Y_n, \end{cases} \tag{21}$$

where Y_b and Y_n denote the benefit and cost types of attributes, respectively.

Step 5 Proposed IVIF-SPC-RS to computing the attribute weight.

Consider that all criteria have different rates of significance. Let $w = (w_1, w_2, ..., w_t)^T$ be the weight set of the criteria set with $\sum_{j=1}^{t} w_j = 1$ and $w_j \in [0,\ 1]$. Here, the criteria weights are computed by combining the objective weights by SPC model and subjective weights by RS model.

Case I Determining of objective weights by the IVIF-SPC model.

The classical SPC weighting model (Gligoric et al. 2023) has developed to assess the criteria weights by means of the characteristic of the criterion that is expressed by its symmetry, i.e., the modulus of symmetry of the criterion to measure its influence on the weights of criteria. The larger value of the modulus denotes the higher weight of a criterion. Here, we extend the SPC model under IVIF setting. The steps are as follows:

Step 5.1 Find the score value of A-IVIFDM.

In this step, we determine the score value of A-IVIFDM by using normalized score function formula

$$\alpha_{ij} = \frac{1}{2}\left(\mathbb{S}\left(\vartheta_{ij}\right) + 1\right), j = 1, 2, ..., t, i = 1, 2, ..., s. \qquad (22)$$

Step 5.2 Determine the symmetry point of each criterion.

In computing the symmetry point of the criterion, only extreme values should be considered. Let $(\alpha_{11},\ \alpha_{21},\ ...,\ \alpha_{i1})^T; \forall\, i$ be a column vector of Y_1 criterion values, with respect to a set of alternatives. If the lower and upper values of the interval $[\beta_1,\ \beta_2]$ are defined as $a = \min(\alpha_{11},\ \alpha_{21},\ ...,\ \alpha_{i1})^T$ and $b = \max(\alpha_{11},\ \alpha_{21},\ ...,\ \alpha_{i1})^T$, respectively, then the point β, which is located in the middle of the interval, represents the symmetry point of criterion Y_1. Thus, the formula for the computation of symmetry point is given as

$$\beta_j = \frac{\min\{\alpha_{ij}\} + \max\{\alpha_{ij}\}}{2}, \forall\, i, j. \qquad (23)$$

Step 5.3 Compute the matrix of absolute distances.

The matrix of absolute distances is computed in the following form:

$$D = \left(|d_{ij}|\right)_{s\times t} = \begin{pmatrix} |\alpha_{11} - \beta_1| & |\alpha_{12} - \beta_2| & \dots & |\alpha_{1t} - \beta_t| \\ |\alpha_{21} - \beta_1| & |\alpha_{22} - \beta_2| & \dots & |\alpha_{2t} - \beta_t| \\ \dots & \dots & \dots & \dots \\ |\alpha_{s1} - \beta_1| & |\alpha_{s2} - \beta_2| & \dots & |\alpha_{st} - \beta_t| \end{pmatrix}_{s\times t}, \qquad (24)$$

where $|\alpha_{ij} - \beta_j|$ denotes the absolute distance from α_{ij} to each symmetry point of criterion β_j.

Step 5.4 Create the matrix of the moduli of symmetry.

Let $D_{i1} = \{d_{11},\ d_{21},\ ...,\ d_{i1}\}, \forall\, i$ be the column vector of the absolute distances concerning criterion Y_1. The modulus for a primal matrix element α_{11} is defined as the ratio of the averaged absolute distance of criterion Y_1 to a value of element α_{11}. Therefore, the matrix of the moduli of symmetry is of the following form:

Table 2 Considered criteria for the present case study

Dimensions	Barriers	References
Economic (*Ec*)	Operation cost per unit (EC-1)	Wu et al. (2013); Lim and Wong (2015); Rani et al. (2020)
	Quality utility value (EC-2)	Wu et al. (2013); Wood (2016); Zhou et al. (2018); Rani et al. (2020)
	Profitability (EC-3)	Lim and Wong (2015); Zhou et al. (2018); Mishra and Rani (2021)
Environmental (*En*)	Resource consumption (EN-1)	Li et al. (2020); Mishra and Rani (2021)
	Energy efficiency (EN-2)	Chen et al. (2015); Lim and Wong (2015), Zhou et al. (2018)
	Pollution & waste production (EN-3)	Lim and Wong (2015); Zhao and Guo (2015); Mishra and Rani (2021)
	Environmental facilities (EN-4)	Kirwan and Wood (2012); Lin et al. (2014); Zhou et al. (2018)
Social (*S*)	Employee turnover rate (S-1)	Zhou et al. (2018); Rani et al. (2020)
	Customer satisfaction (S-2)	Tseng et al. (2014); Lim and Wong (2015); Rani et al. (2020); Mishra and Rani (2021)
	Brand reputation (S-3)	Govindan et al. (2013); Sarkis and Dhavale (2015); Zhou et al. (2018); Rani et al. (2020)
	Local influence degree (S-4)	Zhao and Guo (2015); Zhou et al. (2018)

Table 3 LRs for SRPs in SMEs

LRs	IVIFNs
Extremely high (EH)	([0.90,0.95],[0.0,0.05])
Very high (VH)	([0.80,0.90],[0.05,0.10])
High (H)	([0.70,0.80],[0.10,0.15])
Slightly high (SH)	([0.65,0.70],[0.15,0.25])
Average (A)	([0.55,0.65],[0.20,0.35])
Slightly low (SL)	([0.40,0.50],[0.40,0.45])
Low (L)	([0.25,0.40],[0.45,0.50])
Very low (VL)	([0.15,0.20],[0.60,0.75])
Extremely low (EL)	([0.05,0.10],[0.80,0.90])

$$M = \left(\left|h_{ij}\right|\right)_{s\times t} = \begin{pmatrix} \frac{\sum_{i=1}^{s} d_{i1}}{s\cdot\alpha_{11}} & \frac{\sum_{i=1}^{s} d_{i2}}{s\cdot\alpha_{12}} & \ldots & \frac{\sum_{i=1}^{s} d_{it}}{s\cdot\alpha_{1n}} \\ \frac{\sum_{i=1}^{s} d_{i1}}{s\cdot\alpha_{21}} & \frac{\sum_{i=1}^{s} d_{i2}}{s\cdot\alpha_{22}} & \ldots & \frac{\sum_{i=1}^{s} d_{it}}{s\cdot\alpha_{2n}} \\ \cdots & \cdots & \ldots & \cdots \\ \frac{\sum_{i=1}^{s} d_{i1}}{s\cdot\alpha_{s1}} & \frac{\sum_{i=1}^{s} d_{i2}}{s\cdot\alpha_{s2}} & \ldots & \frac{\sum_{i=1}^{s} d_{it}}{s\cdot\alpha_{st}} \end{pmatrix}_{s\times t}. \tag{25}$$

where $\left|\alpha_{ij} - \beta_j\right|$ denotes the absolute distance from α_{ij} to each symmetry point of criterion β_j.

Step 5.5 Calculate the modulus of symmetry of criterion.

Averaging every column of the previous matrix M, we obtain row vector Q, where every element q represents the modulus of symmetry of the jth criterion. Vector Q is defined as

$$Y_i^{+} = \left(\begin{array}{l}\left[\left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\Omega_p^q(b_{ij}^{-})\right)^{w_j}-1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\Omega_p^q(b_{ij}^{+})\right)^{w_j}-1\right)\right)^{-\frac{1}{q}}\right)^{-1}\right], \\ \left[\left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\mho_p^q(n_{ij}^{-})\right)^{w_j}-1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\mho_p^q(n_{ij}^{+})\right)^{w_j}-1\right)\right)^{\frac{1}{q}}\right)^{-1}\right]\end{array}\right), \tag{30}$$

$$Y_i^{-} = \left(\begin{array}{l}\left[\left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\Omega_p^q(b_{ij}^{-})\right)^{w_j}-1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\Omega_p^q(b_{ij}^{+})\right)^{w_j}-1\right)\right)^{-\frac{1}{q}}\right)^{-1}\right], \\ \left[\left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\mho_p^q(n_{ij}^{-})\right)^{w_j}-1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1+\left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\mho_p^q(n_{ij}^{+})\right)^{w_j}-1\right)\right)^{\frac{1}{q}}\right)^{-1}\right]\end{array}\right), \tag{31}$$

$$Q = (q_j) = \left(\frac{\sum_{i=1}^{s} h_{i1}}{s} \quad \frac{\sum_{i=1}^{s} h_{i2}}{s} \quad \ldots \quad \frac{\sum_{i=1}^{s} h_{it}}{s}\right), \ \forall\, j. \tag{26}$$

Step 5.6 Derive the weights of criteria.

In this step, each objective criterion weight is computed using the vector of moduli of symmetry. The following equation is used for assessing the weights of criteria:

$$w_j^{o} = \left(\frac{q_1}{\sum_{j=1}^{n} q_j} \quad \frac{q_2}{\sum_{j=1}^{n} q_j} \quad \cdots \quad \frac{q_j}{\sum_{j=1}^{n} q_j}\right), \ \forall\, j. \tag{27}$$

Case II Determination of subjective weights by the method of IVIF-RS model.

The RS model (Stillwell et al 1981) has extended under IVIFS context, given as

$$w_j^{s} = \frac{t - r_j + 1}{\sum_{j=1}^{t}\left(t - r_j + 1\right)}, \tag{28}$$

where r_j denotes the rank of jth criterion, $j = 1,2,3,\ldots,t$.

Case III Combined IVIF-SPC-RS weighting model for final weights of criteria.

To obtain the combined weight, DME incorporates the objective and subjective weights of criteria, given as

$$w_j = \varsigma w_j^{o} + (1-\varsigma) w_j^{s}, \tag{29}$$

where $\varsigma \in [0, 1]$ denotes the precision factor of decision strategy.

Step 6 Ratio system model (RSM) using IVIF information.

The IVIF-RSM determines the optimum alternative by using the following procedures:

Step 6.1 Using IVIFIDWA operator, evaluate the significance value of each option by means of benefit and cost criteria.

Step 6.2 Estimate the degree of utilityof each option through Eq. (32).

Table 4 LDM for SRPs by DMEs

Criteria	X_1	X_2	X_3	X_4
Y_1	(L, VL, SL)	(L, L, SL)	(A, VL, VL)	(VL, EL, SL)
Y_2	(SH, H, VH)	(H, H, A)	(A, H, SH)	(SH, SH, VH)
Y_3	(VH, SH, H)	(SH, SL, VH)	(SL, H, VH)	(VH, H, VH)
Y_4	(A, H, VH)	(H, VH, SL)	(VH, A, A)	(VH, SH, A)
Y_5	(H, SH, VH)	(VH, A, H)	(SH, H, SH)	(H, VH, H)
Y_6	(VL, L, VL)	(SL, L, VL)	(VL, VL, A)	(A, L, EL)
Y_7	(VH, SH, SL)	(VH, SH, SH)	(VH, VH, A)	(H, VH, SH)
Y_8	(SH, VH, H)	(SL, H, SH)	(SL, A, H)	(VH, H, SL)
Y_9	(A, SH, VH)	(SL, VH, H)	(SH, H, H)	(SL, SH, A)
Y_{10}	(SH, SH, H)	(VH, H, H)	(SH, SL, VH)	(SH, H, SL)
Y_{11}	(H, SH, H)	(A, VH, VH)	(SL, SL, VH)	(VH, SL, VH)

Table 5 DMEs' weights for SRPs assessment

	g_1	g_2	g_3
LVs	High	Very high	Extremely high
IVIFNs	([0.70,0.80],[0.10,0.15])	([0.80,0.90],[0.05,0.10])	([0.90,0.95], [0.0,0.05])
Weights	0.3092	0.3349	0.3559

Table 6 AIVIF-DM for SRPs

Criteria	X_1	X_2	X_3	X_4
Y_1	([0.278, 0.381], [0.475, 0.552])	([0.307, 0.438], [0.432, 0.482])	([0.302, 0.380], [0.427, 0.593])	([0.221, 0.296], [0.572, 0.665])
Y_2	([0.728, 0.823], [0.089, 0.142])	([0.653, 0.756], [0.128, 0.203])	([0.641, 0.725], [0.143, 0.216])	([0.713, 0.797], [0.101, 0.156])
Y_3	([0.721, 0.815], [0.092, 0.146])	([0.656, 0.759], [0.141, 0.205])	([0.678, 0.793], [0.120, 0.182])	([0.771, 0.874], [0.063, 0.115])
Y_4	([0.706, 0.814], [0.097, 0.169])	([0.665, 0.780], [0.130, 0.194])	([0.650, 0.762], [0.130, 0.238])	([0.678, 0.774], [0.118, 0.197])
Y_5	([0.727, 0.821], [0.090, 0.143])	([0.697, 0.805], [0.102, 0.176])	([0.668, 0.738], [0.131, 0.182])	([0.738, 0.841], [0.079, 0.131])
Y_6	([0.185, 0.273], [0.545, 0.655])	([0.268, 0.372], [0.481, 0.559])	([0.322, 0.404], [0.406, 0.572])	([0.303, 0.413], [0.430, 0.552])
Y_7	([0.643, 0.744], [0.151, 0.215])	([0.706, 0.786], [0.107, 0.161])	([0.733, 0.844], [0.082, 0.156])	([0.723, 0.817], [0.092, 0.145])
Y_8	([0.725, 0.820], [0.090, 0.143])	([0.607, 0.693], [0.177, 0.233])	([0.574, 0.680], [0.194, 0.280])	([0.661, 0.776], [0.132, 0.196])
Y_9	([0.690, 0.787], [0.111, 0.186])	([0.675, 0.789], [0.122, 0.184])	([0.685, 0.773], [0.113, 0.164])	([0.548, 0.629], [0.225, 0.314])
Y_{10}	([0.669, 0.740], [0.130, 0.181])	([0.735, 0.839], [0.081, 0.132])	([0.656, 0.759], [0.141, 0.205])	([0.597, 0.686], [0.186, 0.242])
Y_{11}	([0.684, 0.771], [0.115, 0.165])	([0.743, 0.853], [0.077, 0.147])	([0.594, 0.718], [0.191, 0.263])	([0.711, 0.829], [0.100, 0.165])

$$y_i = \mathbb{S}\left(y_i^+\right) - \mathbb{S}\left(y_i^-\right),\ i = 1, 2, ..., s. \tag{32}$$

Step 7 Reference point model (RPM) using IVIF information.

The IVIF-RPM determines the optimum alternative by using the following procedures:

Step 7.1 Calculate the reference point (RP). The coordinate value of the RP $r_j^* = \left\{r_1^*, r_2^*, ..., r_t^*\right\}$ is an IVIFN r_j^* and is computed using Eq. (33).

$$r_j^* = \left(\left[b_*^-, b_*^+\right], \left[n_*^-, n_*^+\right]\right) = \begin{cases} \left(\left[\max\limits_i b_{ij}^-, \max\limits_i b_{ij}^+\right], \left[\min\limits_i n_{ij}^-, \min\limits_i n_{ij}^+\right]\right), & \text{for benefit criterion } Y_b \\ \left(\left[\min\limits_i b_{ij}^-, \min\limits_i b_{ij}^+\right], \left[\max\limits_i n_{ij}^-, \max\limits_i n_{ij}^+\right]\right), & \text{for cost criterion } Y_n \end{cases}. \tag{33}$$

Step 7.2 Find the divergence from the options to all the coordinates of the RP as

$$D_{ij} = w_j\left(D\left(\tilde{\delta}_{ij}, r_j^*\right)\right), \tag{34}$$

where $D\left(\delta_{ij}, r_j^*\right) = \frac{1}{4\,t\,(\exp(2) - 1)}$

$$\sum_{j=1}^{t}\left[\left(b_{ij}^- - b_*^-\right)\left(\exp\left(\frac{4\,b_{ij}^-}{1 + b_{ij}^- + b_*^-}\right) - \exp\left(\frac{4\,b_*^-}{1 + b_{ij}^- + b_2^-}\right)\right) + \left(b_{ij}^+ - b_*^+\right)\left(\exp\left(\frac{4\,b_{ij}^+}{1 + b_{ij}^+ + b_*^+}\right) - \exp\left(\frac{4\,b_*^+}{1 + b_{ij}^- + b_*^+}\right)\right) + \left(n_{ij}^- - n_*^-\right)\left(\exp\left(\frac{4\,n_{ij}^-}{1 + n_{ij}^- + n_*^-}\right) - \exp\left(\frac{4\,n_*^-}{1 + n_{ij}^- + n_*^-}\right)\right) + \left(n_{ij}^+ - n_*^+\right)\left(\exp\left(\frac{4\,n_{ij}^+}{1 + n_{ij}^+ + n_*^+}\right) - \exp\left(\frac{4\,n_*^+(c_i)}{1 + n_{ij}^+ + n_*^+}\right)\right)\right].$$

Step 7.3 Determine the maximum divergence of each option through Eq. (35).

$$d_i = \max_j D_{ij},\, i = 1(1)s. \tag{35}$$

Step 8 Full multiplicative form model (FMFM) using IVIF information..

The IVIF-FMFM determines the optimum alternative by using the following steps:

Step 8.1 Using IVIFIDWG operator, evaluate the significance value of each option by means of benefit and cost criteria.

$$\alpha_i = \left(\begin{array}{l}\left[\left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\mho_p^q(b_{ij}^-)\right)^{w_j} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\mho_p^q(b_{ij}^+)\right)^{w_j} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}\right],\\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\Omega_p^q(n_{ij}^-)\right)^{w_j} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_b}\left(\Omega_p^q(n_{ij}^+)\right)^{w_j} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}\right]\end{array}\right), \tag{36}$$

$$\beta_i = \left(\begin{array}{l}\left[\left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\mho_p^q(b_{ij}^-)\right)^{w_j} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\mho_p^q(b_{ij}^+)\right)^{w_j} - 1\right)\right)^{\frac{1}{q}}\right)^{-1}\right],\\ \left[\left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\Omega_p^q(n_{ij}^-)\right)^{w_j} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}, \left(1 + \left(\frac{1}{p}\left(\prod_{j\in Y_n}\left(\Omega_p^q(n_{ij}^+)\right)^{w_j} - 1\right)\right)^{-\frac{1}{q}}\right)^{-1}\right]\end{array}\right), \tag{37}$$

where α_i and β_i are IVIFNs.

Step 8.2 Determine the overall utility degree of each option through Eq. (38).

$$u_i = \frac{\mathbb{S}(\alpha_i)}{\mathbb{S}(\beta_i)},\; i = 1, 2, \ldots, s. \tag{38}$$

Step 9 Compute the final ranking of options.

In accordance with vector normalization process, we determine the normalized values of RSM (y_i^*), RPM (d_i^*) and FMFM (u_i^*). Based on Improved Borda rule, the overall assessment degree of each option is determined by

$$I_B(X_i) = y_i^* \cdot \frac{s - \rho\left(y_i^*\right) + 1}{(s(s+1)/2)} - d_i^* \cdot \frac{\rho\left(d_i^*\right)}{(s(s+1)/2)} + u_i^* \cdot \frac{s - \rho\left(u_i^*\right) + 1}{(s(s+1)/2)},\; i = 1(1)s, \tag{39}$$

where $\rho\left(y_i^*\right)$, $\rho\left(d_i^*\right)$ and $\rho\left(u_i^*\right)$ denotes the preference order of RSM, RPM and FMFM, respectively. The larger value of $I_B(U_i)$ denotes the better option.

6 Case study: selection of SRP in SMEs

In the past few decades, the strategies of eco-economy promote the businesses to achieve sustainability and sustainable development goals in SMEs (Zhou et al. 2018; Rani et al. 2020). In this section, we implement the proposed method on a case study of SRP selection in SMEs (adopted from Mishra and Rani 2021), which proves its effectiveness and superiority. Here, we consider X_1, X_2, X_3 and X_4 as four SRP options by means of 11 criteria including three economic, four environmental and four social aspects of sustainability (see Table 2). The detailed description of SRP alternatives in SMEs is given as follows:

Namo e-waste management limited (NEWML) (X_1): It is one of the leading companies of India that recycle electrical and electronic wastes. Successfully operating since 2014, NEWML offers comprehensive, complete and

Table 7 Normalized A-IVIFDM for SRPs

Criteria	X_1	X_2	X_3	X_4
Y_1	([0.475, 0.552], [0.278, 0.381])	([0.432, 0.482], [0.307, 0.438])	([0.427, 0.593], [0.302, 0.380])	([0.572, 0.665], [0.221, 0.296])
Y_2	([0.728, 0.823], [0.089, 0.142])	([0.653, 0.756], [0.128, 0.203])	([0.641, 0.725], [0.143, 0.216])	([0.713, 0.797], [0.101, 0.156])
Y_3	([0.721, 0.815], [0.092, 0.146])	([0.656, 0.759], [0.141, 0.205])	([0.678, 0.793], [0.120, 0.182])	([0.771, 0.874], [0.063, 0.115])
Y_4	([0.706, 0.814], [0.097, 0.169])	([0.665, 0.780], [0.130, 0.194])	([0.650, 0.762], [0.130, 0.238])	([0.678, 0.774], [0.118, 0.197])
Y_5	([0.727, 0.821], [0.090, 0.143])	([0.697, 0.805], [0.102, 0.176])	([0.668, 0.738], [0.131, 0.182])	([0.738, 0.841], [0.079, 0.131])
Y_6	([0.545, 0.655], [0.185, 0.273])	([0.481, 0.559], [0.268, 0.372])	([0.406, 0.572], [0.322, 0.404])	([0.430, 0.552], [0.303, 0.413])
Y_7	([0.643, 0.744], [0.151, 0.215])	([0.706, 0.786], [0.107, 0.161])	([0.733, 0.844], [0.082, 0.156])	([0.723, 0.817], [0.092, 0.145])
Y_8	([0.725, 0.820], [0.090, 0.143])	([0.607, 0.693], [0.177, 0.233])	([0.574, 0.680], [0.194, 0.280])	([0.661, 0.776], [0.132, 0.196])
Y_9	([0.690, 0.787], [0.111, 0.186])	([0.675, 0.789], [0.122, 0.184])	([0.685, 0.773], [0.113, 0.164])	([0.548, 0.629], [0.225, 0.314])
Y_{10}	([0.669, 0.740], [0.130, 0.181])	([0.735, 0.839], [0.081, 0.132])	([0.656, 0.759], [0.141, 0.205])	([0.597, 0.686], [0.186, 0.242])
Y_{11}	([0.684, 0.771], [0.115, 0.165])	([0.743, 0.853], [0.077, 0.147])	([0.594, 0.718], [0.191, 0.263])	([0.711, 0.829], [0.100, 0.165])

Table 8 IVIF-score decision matrix and symmetric point of each criterion

Parameters	X_1	X_2	X_3	X_4	$\min\{\alpha_{ij}\}$	$\max\{\alpha_{ij}\}$	β_j
Y_1	0.592	0.542	0.584	0.699	0.542	0.699	0.620
Y_2	0.830	0.770	0.752	0.813	0.752	0.830	0.791
Y_3	0.825	0.767	0.792	0.867	0.767	0.867	0.817
Y_4	0.814	0.780	0.761	0.784	0.761	0.814	0.787
Y_5	0.829	0.806	0.773	0.842	0.773	0.842	0.808
Y_6	0.685	0.600	0.563	0.566	0.563	0.685	0.624
Y_7	0.755	0.806	0.835	0.826	0.755	0.835	0.795
Y_8	0.828	0.722	0.695	0.777	0.695	0.828	0.762
Y_9	0.795	0.790	0.795	0.660	0.660	0.795	0.727
Y_{10}	0.775	0.840	0.767	0.714	0.714	0.840	0.777
Y_{11}	0.794	0.843	0.714	0.818	0.714	0.843	0.779

Table 9 Resulting matrix of absolute distances for SRPs

Parameters	X_1	X_2	X_3	X_4
Y_1	0.028	0.78	0.036	0.078
Y_2	0.039	0.021	0.039	0.022
Y_3	0.007	0.050	0.025	0.050
Y_4	0.026	0.007	0.026	0.003
Y_5	0.021	0.002	0.035	0.035
Y_6	0.061	0.024	0.061	0.058
Y_7	0.040	0.011	0.040	0.031
Y_8	0.066	0.039	0.066	0.016
Y_9	0.068	0.062	0.068	0.068
Y_{10}	0.002	0.063	0.010	0.063
Y_{11}	0.015	0.064	0.064	0.040

Table 10 Matrix of moduli and criteria weight for SRPs

Parameters	X_1	X_2	X_3	X_4	q_j	w_j^o
Y_1	0.093	0.102	0.095	0.079	0.092	0.155
Y_2	0.037	0.040	0.040	0.037	0.039	0.065
Y_3	0.040	0.043	0.042	0.038	0.041	0.068
Y_4	0.019	0.020	0.021	0.020	0.020	0.033
Y_5	0.028	0.028	0.030	0.027	0.028	0.047
Y_6	0.075	0.085	0.091	0.090	0.085	0.143
Y_7	0.040	0.038	0.036	0.037	0.038	0.063
Y_8	0.057	0.065	0.068	0.060	0.062	0.105
Y_9	0.084	0.084	0.084	0.101	0.088	0.148
Y_{10}	0.045	0.041	0.045	0.048	0.045	0.075
Y_{11}	0.058	0.054	0.064	0.056	0.058	0.097

responsible recycling services. It integrates and best in class technology ensures environmental safety and sustainability while recycling e-waste.

BRP infotech private limited (BRPIPL) (X_2): It is one of the leading computer recycling companies, e-waste recycling companies in India which is involved in the recycling

Table 11 Subjective weight of criteria for SRPs using IVIF-RS method

Criteria	g_1	g_2	g_3	Aggregated IVIFNs	$\mathbb{S}\left(\tilde{\delta}_{kj}\right)$	r_j	w_j^s
Y_1	A	SH	SH	([0.622, 0.685], [0.164, 0.238])	0.726	4	0.1212
Y_2	H	SH	A	([0.635, 0.720], [0.147, 0.223])	0.746	1	0.1667
Y_3	A	A	SH	([0.589, 0.669], [0.181, 0.287])	0.697	6	0.0909
Y_4	H	SH	L	([0.562, 0.661], [0.196, 0.254])	0.694	8	0.0606
Y_5	SH	SL	L	([0.450, 0.544], [0.308, 0.364])	0.581	11	0.0152
Y_6	A	A	H	([0.610, 0.713], [0.156, 0.259])	0.727	3	0.1364
Y_7	H	SL	SL	([0.516, 0.623], [0.261, 0.320])	0.640	9	0.0455
Y_8	A	SH	A	([0.586, 0.668], [0.182, 0.290])	0.696	7	0.0758
Y_9	H	SL	SH	([0.600, 0.686], [0.184, 0.240])	0.716	5	0.1061
Y_{10}	L	H	H	([0.602, 0.719], [0.159, 0.218])	0.736	2	0.1515
Y_{11}	SH	L	SL	([0.453, 0.546], [0.307, 0.363])	0.582	10	0.0303

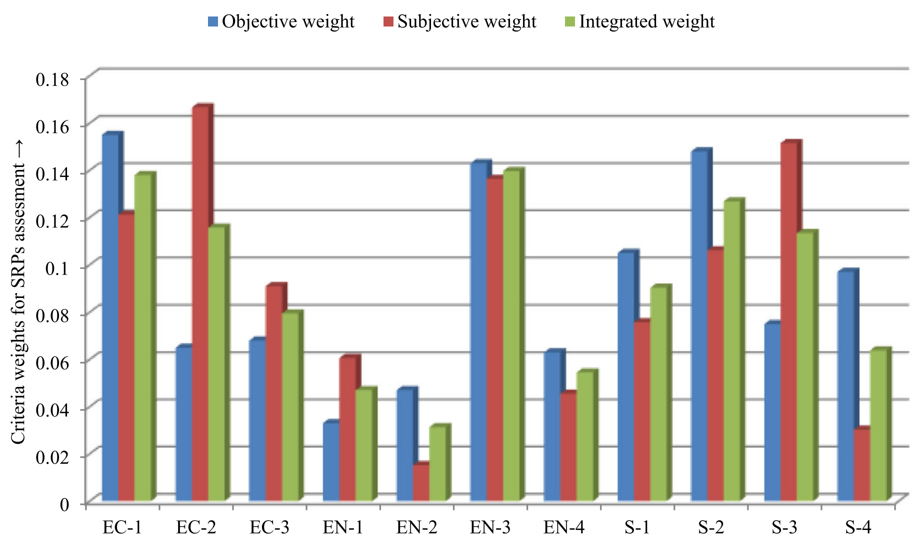

Fig. 2 Combined weight of criteria for SRPs

Table 12 Priority order of SRPs evaluated by RSM

Options	Y_i^+	Y_i^-	s_i^+	s_i^-	s_i	Order
X_1	([0.581, 0.680], [0.197, 0.271])	([0.071, 0.105], [0.829, 0.868])	0.698	0.120	0.579	1
X_2	([0.563, 0.673], [0.211, 0.290])	([0.090, 0.134], [0.804, 0.834])	0.684	0.147	0.537	3
X_3	([0.536, 0.639], [0.237, 0.319])	([0.099, 0.129], [0.784, 0.860])	0.655	0.146	0.509	4
X_4	([0.554, 0.658], [0.222, 0.301])	([0.081, 0.116], [0.823, 0.870])	0.672	0.126	0.546	2

Table 13 Priority order of SRPs evaluated by RPM

	X_1	X_2	X_3	X_4
d_i	0.0011	0.0027	0.0026	0.0029
Ranking	1	3	2	4

of the electronic goods through the state of art recycling process with a vision to protect and conserve the environment by recycling the end-of-life electronics.

E-waste recycle hub (EWRH) (X_3): E-waste Recycle Hub is the leading provider of comprehensive waste management services in India, ensuring services ranging from collection and disposal of waste to recycling and generation of renewable energy. As a pioneer in asset management and electronics recycling, we have been delivering world class services to ensure discarded electronic items are efficiently managed, disposed and recycled.

Table 14 Priority order of SRPs evaluated by the FMFM

Options	A_i	B_i	ς_i	τ_i	u_i	Rank
X_1	([0.772, 0.844], [0.079, 0.123])	([0.662, 0.730], [0.180, 0.228])	0.853	0.746	1.144	1
X_2	([0.755, 0.836], [0.089, 0.136])	([0.707, 0.777], [0.156, 0.185])	0.842	0.786	1.071	3
X_3	([0.733, 0.813], [0.103, 0.157])	([0.724, 0.771], [0.139, 0.215])	0.821	0.785	1.046	4
X_4	([0.741, 0.817], [0.101, 0.150])	([0.687, 0.747], [0.178, 0.231])	0.827	0.756	1.093	2

Table 15 Final preference order of SRPs based on the IVIF-SPC-MULTIMOORA framework

Options	RSM		RPM		FMFM		$I_B(X_i)$	Final ranking
	s_i^*	$\rho(s_i^*)$	d_i^*	$\rho(d_i^*)$	u_i^*	$\rho(u_i^*)$		
X_1	0.533	1	0.222	1	0.525	1	0.401	1
X_2	0.494	3	0.550	3	0.492	3	0.032	3
X_3	0.468	4	0.546	2	0.480	4	− 0.014	4
X_4	0.503	2	0.592	4	0.502	2	0.064	2

Table 16 Weights of diverse criteria for SRPs w. r. t. parameter

ς	ς = 0.0	ς = 0.1	ς = 0.2	ς = 0.3	ς = 0.4	ς = 0.5	ς = 0.6	ς = 0.7	ς = 0.8	ς = 0.9	ς = 1.0
Y_1	0.1212	0.1246	0.1279	0.1313	0.1347	0.1380	0.1414	0.1448	0.1481	0.1515	0.1549
Y_2	0.1667	0.1565	0.1463	0.1361	0.1259	0.1157	0.1055	0.0953	0.0851	0.0749	0.0647
Y_3	0.0909	0.0886	0.0864	0.0841	0.0818	0.0795	0.0773	0.0750	0.0727	0.0704	0.0682
Y_4	0.0606	0.0579	0.0552	0.0524	0.0497	0.0470	0.0443	0.0416	0.0388	0.0361	0.0334
Y_5	0.0152	0.0184	0.0216	0.0248	0.0281	0.0313	0.0345	0.0377	0.0409	0.0441	0.0474
Y_6	0.1364	0.1371	0.1377	0.1384	0.1390	0.1397	0.1403	0.1410	0.1416	0.1423	0.1429
Y_7	0.0455	0.0473	0.0491	0.0509	0.0527	0.0545	0.0563	0.0581	0.0599	0.0617	0.0635
Y_8	0.0758	0.0787	0.0816	0.0845	0.0874	0.0903	0.0932	0.0961	0.0990	0.1019	0.1048
Y_9	0.1061	0.1103	0.1144	0.1186	0.1228	0.1269	0.1311	0.1352	0.1394	0.1436	0.1477
Y_{10}	0.1515	0.1439	0.1362	0.1286	0.1210	0.1134	0.1057	0.0981	0.0905	0.0829	0.0752
Y_{11}	0.0303	0.0370	0.0437	0.0504	0.0571	0.0639	0.0706	0.0773	0.0840	0.0907	0.0974

E-waste recyclers India (EWRI) (X_4): In India, the concept of safe electronic disposal is still in its growing stage. EWRI steps out to fill in that gap. It provides world-class facility in recycling and disposal of e-waste to every Indian, corporate, business houses and general public so when we discard our computer or fax machine, none is harmed.

To choose the best SRP option, a team of three DMEs (g_1, g_2, g_3) has been created. To evaluate the information and data collection for SRP selection in SMEs, the LVs are utilized to define judgments and preferences of DMEs group. Table 3 (adopted from Alrasheedi et al. 2021) represents the LVs and corresponding IVIFNs to evaluate the DMEs and SRP options.

Step 1 The LDM for SRPs assessment by three DMEs is presented in Table 4. The terms g_1, g_2 and g_3 represent the DMEs in related disciplines, Y_1, Y_2,…, Y_{11} signifies the eleven sustainability sub-criteria and X_1, X_2, X_3, X_4 denotes the considered four SRP candidates.

Step 2 Based on the IVIFN scale given in Table 3 and Eq. (19), the weights of DMEs are computed and presented in Table 5 for the performance of SRPs in SMEs.

Step 3 Applying Eq. (20) and Tables 4, 5, the A-IVIFDM $A = \left(\tilde{\delta}_{ij}\right)_{4\times 11}$ with the help of IVIFIDWA (or IVIFIDWG) operator is constructed and shown in Table 6.

Step 4 Since Y_1 and Y_6 are cost criteria and others are benefit types, therefore, we need to make a normalized A-IVIFDM $\mathbb{N} = (\vartheta_{ij})_{4\times 11}$ using Eq. (21) in Table 7.

Step 5 Applying Eqs. (22)–(23), we determine the score value of A-IVIFDM and the symmetry point of each criterion in Table 8. Using Eq. (24), we obtain the absolute

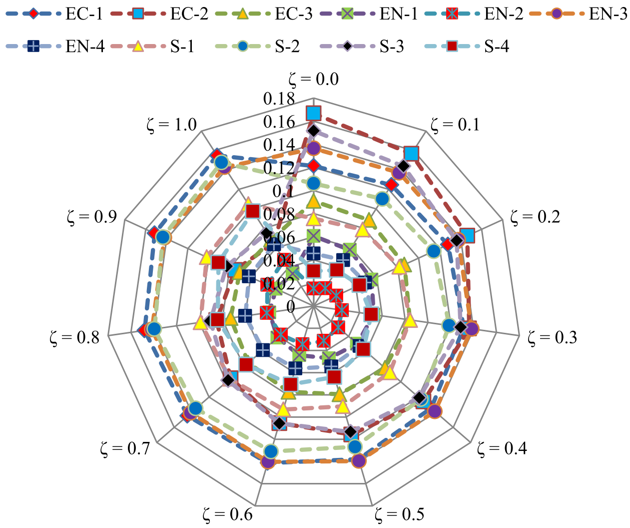

Fig. 3 Changing of criteria weights for SRPs assessment with diverse values of parameter ς

distances matrix in Table 9. Next, we create the matrix of the moduli of symmetry point and the modulus of symmetry of criterion using Eqs. (25)–(26) in Table 10. Finally, we derive the objective weight of criteria using Eq. (27) in Table 10.

Using IVIF-RS model (28), the subjective weights of considered sustainability indicators are computed and shown in Table 11 and Fig. 2.

Based on the objective weights through IVIF-SPC model and subjective weights through IVIF-RS model, the final weights of criteria for $\tau = 0.5$ are shown as (see Fig. 2).w_j = (0.1380, 0.1157, 0.0795, 0.0470, 0.0313, 0.1397, 0.0545, 0.0903, 0.1269, 0.1134, 0.0639).

Here, Fig. 2 shows the weight of different criteria for SRPs. Pollution & waste production (EN-3) with weight value 0.1397 has come out to be the most important criteria for SRPs. Operation cost per unit (EC-1) with weight value 0.138 is the second most significant criteria for SRPs. Customer satisfaction (S-2) has third with weight 0.1269, Quality utility value (EC-2) with weight 0.1157 has fourth, and Brand reputation (S-3) with weight 0.1134 has fifth most important criteria for SRPs, and others are considered crucial criteria for SRPs assessment.

Using Eqs. (30)–(32), the steps of RSM for assessing the SRP options are computed and given in Table 12.

With the use of Eqs. (33)–(35), the steps of RPM for evaluating the SRP options are determined and given in Table 13. $r_j^* = \{([0.221, 0.296], [0.572, 0.665]), ([0.728, 0.823], [0.089, 0.142]), ([0.771, 0.874], [0.063, 0.115]), ([0.706, 0.814], [0.097, 0.169]), ([0.738, 0.841], [0.079, 0.131]), ([0.185, 0.273], [0.545, 0.655]), ([0.733, 0.844], [0.082, 0.156]), ([0.725, 0.820], [0.090, 0.143]), ([0.690, 0.787], [0.111, 0.186]), ([0.735, 0.839], [0.081, 0.132]), ([0.743, 0.853], [0.077, 0.147])\}.r_j^- = \{([0.307, 0.438], [0.432, 0.482]), ([0.641, 0.725], [0.143, 0.216]), ([0.656, 0.759], [0.141, 0.205]), ([0.650, 0.762], [0.130, 0.238]), ([0.668, 0.738], [0.131, 0.182]), ([0.322, 0.404], [0.406, 0.572]), ([0.643, 0.744], [0.151, 0.215]), ([0.574, 0.680], [0.194, 0.280]), ([0.548, 0.629], [0.225, 0.314]), ([0.597, 0.686], [0.186, 0.242]), ([0.594, 0.718], [0.191, 0.263])\}$.

Through Eqs. (36)–(38), the steps of FMFM for evaluating the SRP options are computed and given in Table 14.

Based on Eq. (39), the overall assessment degrees of SRP options are computed and shown in Table 15. Thus, the prioritization order of SRP options is $X_1 \succ X_4 \succ X_2 \succ X_3$. Thus, the optimal SRP alternative is Namo e-waste management limited (NEWML) (X_1) in SMEs in India.

Table 17 Overall assessment degrees of SRPs over different weighting procedures

Weighting procedure	OEDs for considered SRP options			
	X_1	X_2	X_3	X_4
ς = 0.0 (Subjective weight by IVIF-RS)	0.404	0.103	− 0.062	0.027
ς = 0.1	0.403	0.099	− 0.065	0.037
ς = 0.2	0.402	0.094	− 0.068	0.048
ς = 0.3	0.402	0.090	− 0.071	0.060
ς = 0.4	0.401	0.088	− 0.072	0.067
ς = 0.5 (Integrated weight)	0.401	0.032	− 0.014	0.064
ς = 0.6	0.401	0.032	− 0.013	0.062
ς = 0.7	0.401	0.031	− 0.011	0.060
ς = 0.8	0.401	0.031	− 0.009	0.058
ς = 0.9	0.401	0.030	− 0.008	0.056
ς = 1.0 (Objective weight by IVIF-SPC)	0.401	0.030	− 0.006	0.054

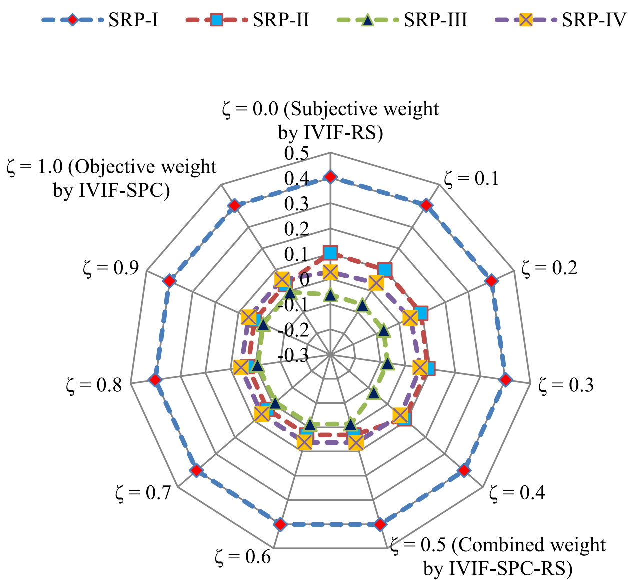

Fig. 4 Sensitivity investigation of overall assessment degrees of SRPs with diverse values of parameter ς

6.1 Sensitivity investigation

In this section, we firstly conduct a sensitivity investigation concerning different values of parameter ς. The first case determines the weights of considered criteria when ς = 0.0 to ς = 1.0 in place of integrated weighting technique. The assessment weight of criteria for SRPs assessment is calculated and shown in Table 16 and Fig. 3 for ς = 0.0 to

Table 20 OCDs for SRPs assessment

Options	$Q_i^{(1)}$	$Q_i^{(2)}$	$Q_i^{(3)}$	Q_i	Ranking
X_1	0.2586	2.1326	1.0000	1.9504	1
X_2	0.2491	2.0541	0.9633	1.8787	3
X_3	0.2425	2.0000	0.9379	1.8292	4
X_4	0.2499	2.0612	0.9666	1.8852	2

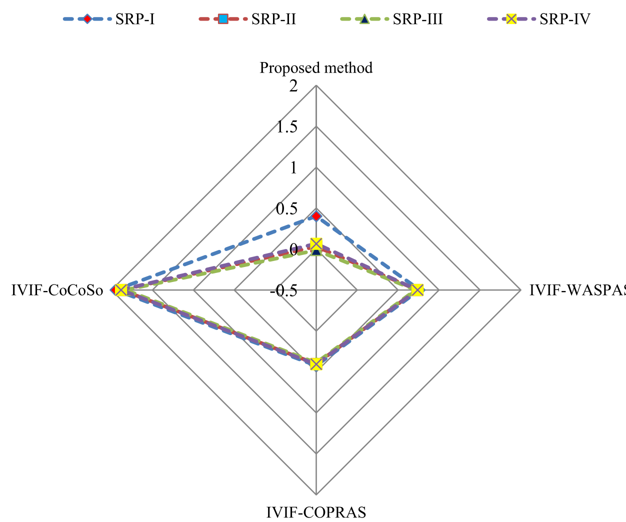

Fig. 5 Comparison of proposed with extant methods for SRPs assessment

Table 18 Computational outcomes of IVIF-WASPAS model

Options	WSM		WPM		UD	Rank
	$C_i^{(1)}$	$\mathbb{S}\left(C_i^{(1)}\right)$	$C_i^{(2)}$	$\mathbb{S}\left(C_i^{(2)}\right)$		
U_1	([0.657, 0.753], [0.130, 0.198])	0.770	([0.640, 0.733], [0.144, 0.215])	0.753	0.7619	1
U_2	([0.631, 0.733], [0.149, 0.225])	0.748	([0.607, 0.697], [0.171, 0.252])	0.720	0.7339	3
U_3	([0.600, 0.717], [0.171, 0.246])	0.725	([0.574, 0.700], [0.192, 0.266])	0.704	0.7146	4
U_4	([0.633, 0.737], [0.153, 0.225])	0.748	([0.610, 0.711], [0.174, 0.249])	0.725	0.7364	2

Table 19 Results of IVIF-COPRAS framework

Options	$\wp_i$	$\mathbb{S}(\wp_i)$	$\Im_i$	$\mathbb{S}(\Im_i)$	ℓ_i	U_i
X_1	([0.581, 0.680], [0.197, 0.271])	0.698	([0.071, 0.105], [0.829, 0.868])	0.120	0.4243	100.0
X_2	([0.563, 0.673], [0.211, 0.290])	0.684	([0.090, 0.134], [0.804, 0.834])	0.147	0.4031	95.00
X_3	([0.536, 0.639], [0.237, 0.319])	0.655	([0.099, 0.129], [0.784, 0.860])	0.146	0.3890	91.69
X_4	([0.554, 0.658], [0.222, 0.301])	0.672	([0.081, 0.116], [0.823, 0.870])	0.126	0.4075	96.04

ς = 1.0. For ς = 0.0, the performance of criteria for SRPs is given in first column of Table 15 and the quality utility value (EC-2) and brand reputation (S-3) with a weight of values 0.1667 and 0.1515 have come out to be the most important criteria for SRPs, respectively. For ς = 1.0, the performance of criteria for SRPs is given in last column of Table 16 and the operation cost per unit (EC-1) and customer satisfaction (S-3) with a weight of values 0.1550 and 0.1480 have come out to be the most important criteria for SRPs, respectively. Using the IVIF-SPC-based method, the overall assessment degrees and ranking of SRPs are computed and given in Table 17 and Fig. 4. The overall assessment degrees of SRP alternatives are $I_B(X_1)$ = 0.401, $I_B(X_2)$ = 0.030, $I_B(X_3)$ = − 0.006 and $I_B(X_4)$ = 0.054, and the preference ranking of SRPs is $X_1 \succ X_4 \succ X_2 \succ X_3$. Applying the IVIF-RS model, the overall assessment degrees of SRP alternatives are $I_B(X_1)$ = 0.404, $I_B(X_2)$ = 0.103, $I_B(X_3)$ = − 0.062 and $I_B(X_4)$ = 0.027, and the preference ranking of SRPs is $X_1 \succ X_4 \succ X_2 \succ X_3$. It implies that the Namo e-waste management limited (NEWML) SRP-I (X_1) is the best SRP choice among others, whereas an alternative SRP-III (X_3) is the least preferable SRP choice for each value of parameter ς.

6.2 Comparative analysis

In this part of the study, the IVIF-SPC-RS-MULTIMOORA methodology is compared some of the prior developed approaches under IVIF environment. We have selected those approaches that have good effectiveness in handling the MCDM problems with multiple experts, given as follows: IVIF-WASPAS (Mishra and Rani 2018), IVIF-COPRAS (Wang et al. 2016) and IVIF-CoCoSo (Alrasheedi et al. 2021).

6.2.1 IVIF-WASPAS

Steps 1–4 Follow the same process as given in Sect. 5.

Step 5 Calculate the measures of WSM and WPM for *i*th alternative using Eqs. (40) and (41), respectively.

$$C_i^{(1)} = \underset{j=1}{\overset{t}{\oplus}} w_j\, \vartheta_{ij}. \tag{40}$$

$$C_i^{(2)} = \underset{j=1}{\overset{t}{\otimes}} \vartheta_{ij}^{w_j}. \tag{41}$$

Step 6 Determine the measure of WASPAS for each option.

$$C_i = \lambda\, C_i^{(1)} + (1 - \lambda)\, C_i^{(2)}. \tag{42}$$

Step 8 In accordance with the WASPAS measures, estimate the ranking order of the options.

For the given case study of SRPs assessment, the overall results of IVIF-WASPAS method are computed using Eqs. (40)–(42), presented in Table 18 (for λ = 0.5). The preference ordering of SRP options is $X_1 \succ X_4 \succ X_2 \succ X_3$ for $\lambda = 0.5$. Thus, the most appropriate SRP alternative is Namo e-waste management limited (NEWML) SRP-I (X_1).

6.2.2 IVIF-COPRAS

Steps 1–4 Follow the steps of IVIF-COPRAS.

Step 5 Add the criteria values for benefit and cost.

In IVIF-COPRAS method, each candidate is assessed with its sums of maximizing criteria considered as benefit-type and minimizing criteria considered as cost-type, presented as

$$\wp_i = \underset{j=1}{\overset{l}{\oplus}} w_j\, \delta_{ij}, \tag{43}$$

$$\Im_i = \underset{j=l+1}{\overset{t}{\oplus}} w_j\, \delta_{ij}. \tag{44}$$

Step 6 Estimate the relative degree of each alternative using Eq. (45).

$$\ell_i = \imath\, \mathbb{S}(\wp_i) + (1 - \imath)\frac{\min_i \mathbb{S}(\Im_i) \sum_{i=1}^{s} \mathbb{S}(\Im_i)}{\mathbb{S}(\wp_i) \sum_{i=1}^{s} \frac{\min_i \mathbb{S}(\Im_i)}{\mathbb{S}(\Im_i)}};\ i = 1, 2, \ldots, s, \tag{45}$$

Step 7 Assess the utility degree of each option using

$$U_i = \frac{\ell_i}{\ell_{\max}} \times 100\,\%, \tag{46}$$

The results of the IVIF-COPRAS method for SRPs assessment are obtained using Eqs. (43)–(46), shown in Table 19 ($\imath = 0.5$). Thus, the preference ordering of the alternatives is $X_1 \succ X_4 \succ X_2 \succ X_3$ and the option Namo e-waste management limited (NEWML) SRP-I (X_1) is considered to be best SRP alternative in SMEs.

6.2.3 IVIF-CoCoSo

Steps 1–5 Similar to the IVIF-WASPAS model.

Step 6 Estimate the “balanced compromise degrees (BCDs)” of options as

$$Q_i^{(1)} = \frac{\mathbb{S}\left(S_i^{(1)}\right) + \mathbb{S}\left(S_i^{(2)}\right)}{\sum_{i=1}^{s}\left(\mathbb{S}\left(S_i^{(1)}\right) + \mathbb{S}\left(S_i^{(2)}\right)\right)}, \tag{47}$$

$$Q_i^{(2)} = \frac{\mathbb{S}\left(S_i^{(1)}\right)}{\min\limits_i \mathbb{S}\left(S_i^{(1)}\right)} + \frac{\mathbb{S}\left(S_i^{(2)}\right)}{\min\limits_i \mathbb{S}\left(S_i^{(2)}\right)}, \tag{48}$$

$$Q_i^{(3)} = \frac{\vartheta\,\mathbb{S}\left(S_i^{(1)}\right) + (1-\vartheta)\mathbb{S}\left(S_i^{(2)}\right)}{\vartheta\,\max\limits_i \mathbb{S}\left(S_i^{(1)}\right) + (1-\vartheta)\max\limits_i \mathbb{S}\left(S_i^{(2)}\right)}, \tag{49}$$

Step 7 Assess the "overall compromise degree (OCD)" of options are computed as

$$Q_i = \left(Q_i^{(1)} Q_i^{(2)} Q_i^{(3)}\right)^{1/3} + \frac{1}{3}\left(Q_i^{(1)} + Q_i^{(2)} + Q_i^{(3)}\right). \tag{50}$$

Step 8 Prioritize the options using OCD (Q_i) in decreasing order.

For the aforesaid SRP selection problem, the OCDs are computed using Eqs. (47)–(50) and shown in Table 20. From Table 20, the Namo e-waste management limited (NEWML) SRP-I (X_1) is the best SRP among others.

Table 15 and Tables 18, 19, 20 present the ranking orders of four SRP options by different approaches and it is graphically depicted in Fig. 5. From Fig. 5, it can easily be noticed that the optimal SRP choice is Namo e-waste management limited (NEWML) SRP-I (X_1) using all the MCDA approaches, namely CoCoSo, WSM, WASPAS and COPRAS.

The main advantages of the developed IVIF-SPC-RS-MULTIMOORA methodology are as follows:

- The IVIF-WASPAS (Mishra and Rani 2018) and the IVIF-CoCoSo method combine the WSM and WPM to increase the accuracy of results. The WASPAS method uses the concept of algebraic operators for determining WSM and WPM. The IVIF-COPRAS model uses the concept of averaging algebraic operators, while the proposed IVIF-SPC-RS-MULTIMOORA method employs the IVIFIDWA and IVIFIDWG operators, which are the generalized version of algebraic operators and proposed IVIF-divergences measure to find the reference point of each SRP option. Thus, the proposed IVIF-SPC-RS-MULTIMOORA method provides a better result than the extant methods.
- The proposed method integrates the objective weights by IVIF-SPC-based process and subjective weights by IVIF-RS model to determine the final weights of criteria for SRPs assessment. The weighting procedure developed in this study overcomes the drawbacks of simply using objective weighting procedure with the combination of divergence measure and entropy (Mishra and Rani 2018) and subjective weighting procedure with ANP model (Wang et al. 2016) and combined weights of criteria with similarity measure (Alrasheedi et al. 2021).
- The IVIF-SPC-MULTIMOORA model considers the both types of criteria according to the overall assessment degrees evaluation, which offers more accurate information as compared to different previously developed approaches mostly taking only the benefit or cost criteria into account. Consequently, the standards are found practicable by the IVIFIDWA and IVIFIDWG operators, which is more accurate in the sense that the expert knowledge not only about the IVIF-IS performance of SRPs over the criteria but also a relative comparison of the performances among them. Thus, the proposed IVIF-SPC-RS-MULTIMOORA model is more consistent and has good stability with the previously developed methodologies.

7 Conclusions

As a generalized form of IFSs, IVIFSs have been taken into consideration in this study to address the uncertainty and imprecision related to MCDA problems. The arithmetic AOs have several limitations and lack of flexible parameters. This inspired us to introduce some innovative operational laws for IVIFNs using improved Dombi operations. For this purpose, we have created two AOs as the IVIFIDWA and IVIFIDWG operators with some key properties, including shift invariance, idempotency, boundedness and monotonicity. Second, new IVIF-DM is discussed with some elegant properties. Third, a new criteria weighting approach is extended with the IVIF-SPC and IVIF-RS models. Fourth, an integrated IVIF-SPC-RS-MULTIMOORA model is presented to deal with MCDA problem under IVIF environment. Finally, we take a case study to choose the best SRP alternative concerning multiple sustainability indicators in SMEs. The outcome of the developed model is discussed that the "Namo e-waste management limited (NEWML)" (SPR-1) as the best SRP in SMEs in India. By examining the sensitivity of the weighted criteria, we were able to demonstrate the viability of the suggested approach. We can indicate that the presented decision framework can be applied successfully in MCDA problems in IVIFSs scenario. The findings of this work prove that the present methodology has a great significance and robustness and is well-consistent than the prior introduced IVIF-MULTIMOORA, IVIF-TOPSIS, IVIF-WASPAS, IVIF-COPRAS and IVIF-CoCoSo approaches.

The certain limitations of the developed framework are important to be aware of. In this context, we present the limitations of the introduced MCDA methodology as follows: (1) In the developed IVIF-SPC-RS-MULTIMOORA method, the single normalization process is applied for assessing the SRP in SMEs, (2) In the developed IVIF-SPC-RS-MULTIMOORA method, the selection of DMEs is limited. However, in realistic circumstances, there is requirement to consider the large number of DMEs for assessment of SRP selection, (3) As evaluation of SRPs selection problem becomes increasingly serious, more dimensions of sustainability should be considered in the assessment of SRP in SMEs.

In the future, new MCDA models with diverse integrated tools can be established for determining a practical solution to the decision analysis problems, such as selection of electric vehicle charging station location, the treatment technology for medical waste, the choice of a technological forecasting method, the problem of choosing a cloud vendor and others. Additionally, it is possible to build new IVIF information measures-based weighting tools to determine the weights of the DMEs.

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Data availability The data used to support the findings of this study are included within this article. However, the reader may contact the corresponding author for more details on the data.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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Authors and Affiliations

Pratibha Rani[1] · Arunodaya Raj Mishra[2] · Dragan Pamucar[3] · Jabir Ali[4] · Ibrahim M. Hezam[5]

✉ Dragan Pamucar
dpamucar@gmail.com

Pratibha Rani
pratibha138@gmail.com

Arunodaya Raj Mishra
arunodaya87@outlook.com

Jabir Ali
jabirali.cse@gmail.com

Ibrahim M. Hezam
ialmishnanah@ksu.edu.sa

[1] Department of Engineering Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Andhra Pradesh 522302, India

[2] Department of Mathematics, Government College Raigaon, Satna, Madhya Pradesh 485441, India

[3] Faculty of Organizational Sciences, College of Engineering, University of Belgrade, Yuan Ze University Belgrade, Belgrade, Serbia, Taiwan

[4] School of Engineering and Technology, Department of Computer Science and Engineering, Sharda University, Greater Noida, UP, India

[5] Department of Statistics and Operations Research, College of Sciences, King Saud University, Riyadh, Saudi Arabia