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APPLICATION OF SOFT COMPUTING

Investigation of the brain carcinoma based on generalized variation coefficient similarity measures using complex q-rung orthopair fuzzy information

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Abstract
Brain carcinoma is one of the massive dangerous diseases in the human body, and certain intellectuals have been affected by them. Additionally, by using the complex q-rung orthopair fuzzy set, which is the massive important, and dominant technique to manage indeterminate and ambiguous information in genuine life troubles. This study aims to employ the principle of variation coefficient similarity measures and generalized variation coefficient similarity measures under the complex q-rung orthopair fuzzy sets and illustrated their properties. Certain special cases of the elaborated measures are investigated to expand the superiority of the investigated works. Moreover, by using the presented generalized variation coefficient similarity measures under the complex q-rung orthopair fuzzy information, a medical diagnosis is illustrated to determine the most dangerous sorts of brain carcinoma in the human body to determine the supremacy and dominance of the elaborated measures. Lastly, certain examples are illustrated based on proposed measures under a complex q-rung orthopair fuzzy set to find the advantages and sensitive analysis of the initiated measures to illustrate the rationality and dominance of the developed measures.

Keywords Complex q-rung orthopair fuzzy sets · Variation coefficient similarity measures · Generalized variation coefficient similarity measures · Brain cancer · Decision-making strategy

Abbreviations

FS	Fuzzy sets
IFS	Intuitionistic fuzzy sets
PFS	Pythagorean fuzzy sets
q-ROFS	Q-rung orthopair fuzzy sets
CFS	Complex fuzzy sets
CIFS	Complex intuitionistic fuzzy sets
CPFS	Complex Pythagorean fuzzy sets
Cq-ROFS	Complex q-rung orthopair fuzzy sets

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VCSM	Variation coefficient similarity measures
HVSM	Hamming variation similarity measures
HWVSM	Hamming weighted variation similarity measures
GHVSM	Generalized hamming variation similarity measures
GHWVSM	Generalized hamming weighted variation similarity measures

1 Introduction

The important strategy if you do not know weirdly as it appears, tossing a coin is a genuine decision-making strategy. Continually verifying is the easiest of the proposed strategies, and if utilized suitably it can be valuable and time-saving. A multi-attribute decision-making strategy is one of the skillful and valuable parts of the decision-making strategy used for the selecting of the beneficial term from the family of terms, called alternatives. A massive part of implementations has been described in the deliberation of crisp set theory. Ambiguity and complexity have still involved in many practical problems, and due to this reason, a lot of data have been lost during decision-making processes in the presence of crisp set theory. To reduce the loss of data during decision-making processes, Zadeh (1965) invited a new theory in the circumstance of abstract algebra, called FS by improving the range of the crisp set which is {0, 1} into [0,1]. They have created a lot of possibilities for a decision-maker to make a decision instead of two possibilities such as "0" or "1." A huge number of scholars have utilized the theory of fuzzy set in the region of many different fields. But handling two sorts of data in the shape of one term have faced by different intellectuals, and in the presence of FS has not been able to invent the solution to the above problem, for this, Atanassov (1986) simplified the above dilemmas by providing a new and well-known technique, called IFS. Atanassov put two different terms in one set and gave their new characteristic in the term: $0 \leq \zeta_A'(\Phi) + \eta_A'(\Phi) \leq 1$. Several well-known and high-level profile scholars have worked on it, for example, Dengfeng and Chuntian (2002), Dengfeng (2004), Garg and Kumar (2018), Wang and Xin (2005), Xu and Chen (2008), Liu and Chen (2016), Liu et al. (2014), Garg (2016a, 2017a, 2016b), Campaigner et al. (2020), and Dengfeng (2005). Some problems occurred in the shape of interval values, for instance, if an expert suggested their opinion, Pakistan will be doing score between 170 and 190 against India in the T-20 World Cup 2022. Such sort of situations cannot be handled from IFS, for handling such sort of challenging scenario, Atanassov (1999) diagnosed the concept of interval-valued IFS (IVIFS), by characterizing the duplet into interval-valued (IV). IVIFS has the modified technique of IV fuzzy sets (IVFS), founded by Zadeh (1975). Several well-known and high-level profile scholars have worked on it, for example, Kumar and Garg (2018), Wei et al. (2011), Xu (2018), Xu (2010), Yue (2011), Garg (2016c), and Liu (2017, 2013).

Yager (2013) further extended the methodology of IFS to invent the PFS with a novel suggestion in the term of condition: $0 \leq \zeta_A'^2(\Phi) + \eta_A'^2(\Phi) \leq 1$. To reduce genuine life ambiguity, some people have diagnosed some applications, for illustration, Garg (2016d, 2017b). Moreover, the fundamental theory of IVPFS was invented in Garg 2017c, and their applications have been employed in Wei and Lu (2017) and Wei and Lu (2018). Additionally, Yager (2016) again thinks about the improvement of the technique of PFS to diagnose the q-ROFS. The prevailing drawbacks of IFS and PFS have generalized the last version of q-ROFS, and it is the massively improved version of the existing drawbacks due to the mathematical shape of the q-ROFS. In the presence of the above discussions, we get the value and importance of the q-ROFS and describe their application in the shape, for instance, Wei et al. (2018), Liu and Wang (2018a), Peng et al. (2018), Liu and Liu (2018), Liu and Wang (2018b), Wei et al. (2019), Akram et al. (2021), Akram and Shahzadi (2021), and Liu et al. (2021a, 2021b).

Ramot et al. (2013) invented a new version of FS in the shape of CFS by extending the truth grade into a complex-valued truth grade, whose real and unreal terms lie between unit intervals. Furthermore, Alkouri and Salleh (Ramot et al. 2002) exposed the CIFS and the conception of complex intuitionistic fuzzy soft sets, diagnosed by Kumar and Bajaj (2014). Complex intuitionistic fuzzy relations were founded by Alkouri and Salleh (2012), and several measures and operators were exposed in Rasoulzadeh et al. (2022); Das and Granados (2022); Garg and Rani (2019a); Rani and Garg (2018); and Garg and Rani (2019b). Further, Ullah et al. (2020a) diagnosed a novel framework of CPFS. In the presence of CIFS and CPFS, experts faced a lot of dilemmas, because the prevailing theories have a lot of weaknesses in their mathematical structure, for illustration, if some individual give their opinion in the term $0.9e^{i2\pi(0.2)}$ for TG and $0.7e^{i2\pi(0.8)}$ for FG, then such that $0.9 + 0.7 = 1.6 \geq 1, 0.2 + 0.8 = 1 \geq 1$, and $0.9^2 + 0.7^2 = 0.81 + 0.49 = 1.30 \geq 1, 2^2 + 0.8^2 = 0.04 + 0.64 = 0.68 \in [0, 1]$, in the consideration of the above results, the resultant value of the real parts cannot lie in the unit interval. By reducing the complications in the decision-making procedure, the Cq-ROFS was diagnosed by Liu et al. (2020a), by including their well-known technique, which stated that the sum of the q-powers of the real part (also for the unreal part) of the duplets is lies with in-unit interval. Several well-known and high-level profile scholars have worked on it, which are diagnosed in Ali and Mahmood (2020); Mahmood and Ali (2021); and Mahmood and Ali (2020).

One of the most important and dangerous diseases which occurred in the body of many unlucky peoples, called brain cancer, is a type of disease that arises in the brain tissues. Brain cancer works in the different cells and functions of the human brain like muscle control, sensation, memory, and other parts of the normal body functions. If someone is diagnosed the brain cancer in the initial stages, then the treatment of brain cancer is possible in the consideration of some brain surgery, with the help of radiation and chemotherapy. A lot of times, some affected people have needed the combination of these therapies. These all processes are depended on the stages, types, locations, size of the tumor, health, and age of the patients, affected by brain cancer. A lot of practical applications have been diagnosed in Alzubi et al. (2019) and Alzubi (2016). In upcoming times, a lot of ideas (Alzubi et al. 2018; Sethuraman et al. 2019; Campagner and Ciucci 2017; Cabitza and Campagner 2021; Hernandez-Boussard et al. 2020) can use for the finding of brain cancer. But it is very difficult to demonstrate the most dangerous types of brain cancer, and how we find their symptoms in the consideration of some prevailing theories. The advantage of the Cq-ROFS is that they can easily handle awkward and complicated data in genuine life troubles. For illustration, an enterprise wants to establish a new campus of luxurious cars. The main aims of this campus are to provide particular data about each car in the form of: (i) the name of the car and their comfort zone and (ii) the price of the car and its warranty. From the above data, we noted that each point covers two sorts of data in the shape of one term. For handling such problematic situations, the existing theories are enabled to determine their solution. For handling the above cases, the theory of Cq-ROFS is massively feasible and dominant to find the supremacy and accuracy of the invented works. In the presence of the following queries, we demonstrated some questions, described here:

1. How do we explore the most excellent optimum in the attention of MADM skills?
2. How do we collect the group of Cq-ROFNs into a single element?
3. How do we employ some measures in the attention of Cq-ROFNs and investigate their relationships?
4. How do we find the most dangerous type of brain cancer?
5. How do we find their solution in the presence of the invented works?

Based on the previous discussions, we obtained that the invented works for Cq-ROFS are suitable to investigate their solutions, several major aspects are described below:

1. To explore the most excellent optimum in the attention of MADM skills, we use to explore some new measures for Cq-ROFNs.
2. To collect the group of Cq-ROFNs into a single element, we propose the VCSM.
3. To employ some measures in the attention of Cq-ROFNs and investigate their relationships with the help of some practical examples.
4. To find the most dangerous type of brain cancer, we consider some data and resolve it by using the invented measures.
5. To find the supremacy of the invented works, we discuss the advantages and comparative analysis of the invented works.

From the above analysis, it is clear that many scholars have proposed different types of measures based on fuzzy sets and their extensions, but no one can think about the main idea of variation coefficient similarity measures based on Cq-ROFS because the theory of variation coefficient similarity measures is the modified version of many similarity measures. Some advantages and beneficial points of the Cq-ROFS have been described here:

1. We get the conception of IFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ with $0 \leq \zeta_A(\Phi) + \eta_A(\Phi) \leq 1$.
2. We get the conception of PFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ with $0 \leq \zeta_A^2(\Phi) + \eta_A^2(\Phi) \leq 1$.
3. We get the conception of q-ROFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ with $0 \leq \zeta_A^q(\Phi) + \eta_A^q(\Phi) \leq 1, q \geq 1$.
4. We get the conception of CIFS when $0 \leq \zeta_A(\Phi) + \eta_A(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \leq 1$.
5. We get the conception of CPFS when $0 \leq \zeta_A^2(\Phi) + \eta_A^2(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \leq 1$.
6. The conception of IFS, PFS, q-ROFS, CIFS, and CPFS is the specific cases of the Cq-ROFS, due to these all reasons, we considered it for the invented works.

This article characterized several stages in the shape: Sect. 2 talks about several prevailing bits of knowledge such as PFSs, CPFSs, q-ROFSs, VCSM, and their fundamental results. Section 3 stated about VCSMs, generalized VCSMs, and their studied remarkable cases. The properties and proofs of the proposed methods are also discussed in detail. Various examples are illustrated in the consideration of the invented works for Cq-ROFS to diagnose the advantages and sensitive analysis of the initiated measures to exemplify the rationality and power of the developed measures, as stated in Sect. 4. Section 5 talked about the conclusion of this article. Finally, we added a new figure to show the general procedures of the proposed method, see Fig. 1.

2 Preliminaries

A lot of well-known exiting theories are recalled in the form of PFSs, CPFSs, q-ROFSs, and their fundamental properties. Several mathematical symbols $X, \zeta_A^{'}(\Phi)$, and $\eta_A^{'}(\Phi)$ stated the fixed set, TG and FG.

Definition 1 Yager (2013) A PFS A on finite universal set X is given by:

$$A = \left\{ \left(\zeta_A^{'}(\Phi), \eta_A^{'}(\Phi) \right) : \Phi \in X \right\} \tag{1}$$

with $0 \leq \zeta_A^{'\,2}(\Phi) + \eta_A^{'\,2}(\Phi) \leq 1$. Further, the hesitancy degree of PFS follows as: $\xi_A = \left(1 - \zeta_A^{'\,2}(\Phi) - \eta_A^{'\,2}(\Phi) \right)^{\frac{1}{2}}$. The Pythagorean fuzzy number (PFN) is represented by $A = \left(\zeta_A^{'}, \eta_A^{'} \right)$.

Definition 2 Ullah et al. (2020a) A CPFS A on finite universal set X is given by:ξ_A^q

$$A = \left\{ \left(\zeta_A^{'}(\Phi), \eta_A^{'}(\Phi) \right) : \Phi \in X \right\} \tag{2}$$

where $\zeta_A^{'}(\Phi) = \zeta_A(\Phi) e^{i2\pi\varphi_{\zeta_A}(\Phi)}$ and $\eta_A^{'}(\Phi) = \eta_A(\Phi) e^{i2\pi\varphi_{\eta_A}(\Phi)}$ stated the Tg and FG with $0 \leq \zeta_A^2(\Phi) + \eta_A^2(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \leq 1$. The hesitancy degree of a CPFS follows as: $\xi_A = \left(1 - \zeta_A^2(\Phi) - \eta_A^2(\Phi)\right)^{\frac{1}{2}} e^{i2\pi\left(1 - \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi)\right)^{\frac{1}{2}}}$. The complex Pythagorean fuzzy number (CPFN) is represented by $A = \left(\zeta_A(\Phi) e^{i2\pi\varphi_{\zeta_A}(\Phi)}, \eta_A(\Phi) e^{i2\pi\varphi_{\eta_A}(\Phi)} \right)$.

Definition 3 Yager (2016) A q-ROFS A on finite universal set X is given by:

$$A = \left\{ \left(\zeta_A^{'}(\Phi), \eta_A^{'}(\Phi) \right) : \Phi \in X \right\} \tag{3}$$

with $0 \leq \zeta_A^{'\,q}(\Phi) + \eta_A^{'\,q}(\Phi) \leq 1$. Further, the hesitancy degree of a PFS is given as follows: $\xi_A = \left(1 - \zeta_A^{'\,q}(\Phi) - \eta_A^{'\,q}(\Phi) \right)^{\frac{1}{q}}$. The q-rung orthopair fuzzy number (q-ROFN) is represented by $A = \left(\zeta_A^{'}, \eta_A^{'} \right)$.

Definition 4 Yager (2016) Let $A = \left(\zeta_A^{'}, \eta_A^{'} \right)$ and $B = \left(\zeta_B^{'}, \eta_B^{'} \right)$ are two q-ROFNs. Then,

1. $A \subseteq B$ iff $\zeta_A^{'} \leq \zeta_B^{'}$ and $\eta_A^{'} \geq \eta_B^{'}$;
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
3. $A^c = \left(\eta_A^{'}, \zeta_A^{'} \right)$.

Definition 5 Pramanik et al. (2017) The variation coefficient similarity measure between two vectors $X = (\Phi_1, \Phi_2, \ldots, \Phi_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ is given by:

$$V(X, Y) = \theta \frac{2XY}{\|X\|^2 + \|Y\|^2} + (1 - \theta) \frac{XY}{\|X\| + \|Y\|} \tag{4}$$

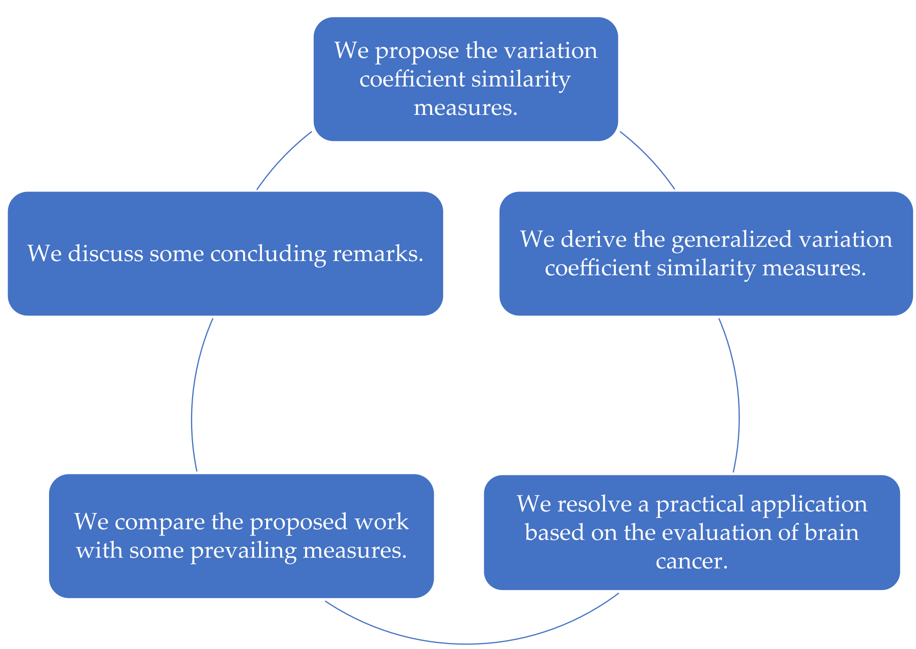

Fig. 1 Geometrical representation of the proposed work

$$= \theta \frac{\sum_{i=1}^{n} 2\Phi_i y_i}{\sum_{i=1}^{n} \Phi_i^2 + \sum_{i=1}^{n} y_i^2} + (1-\theta) \frac{\sum_{i=1}^{n} \Phi_i y_i}{\sqrt{\sum_{i=1}^{n} \Phi_i^2} + \sqrt{\sum_{i=1}^{n} y_i^2}} \quad (5)$$

It satisfies the following conditions:

1. $0 \le V(X,Y) \le 1$;
2. $V(X,Y) = V(Y,X)$;
3. $V(X,Y) = 1$ iff $\Phi_i = y_i,\ i = 1,2,..,n$.

Definition 6 Liu et al. (2020a) A Cq-ROFS A on finite universal set X is given by: ζ_A^q

$$A = \left\{ \left(\zeta_A^{'}(\Phi), \eta_A^{'}(\Phi) \right) : \Phi \in X \right\} \quad (6)$$

with $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$. The hesitancy degree of a Cq-ROFS follows as: $\xi_A = \left(1 - \zeta_A^q(\Phi) - \eta_A^q(\Phi)\right)^{\frac{1}{q}} e^{i2\pi\left(1-\varphi_{\zeta_A}^q(\Phi)+\varphi_{\eta_A}^q(\Phi)\right)^{\frac{1}{q}}}$. The complex q-rung orthopair fuzzy number (Cq-ROFN) is represented by $A = \left(\zeta_A(\Phi)e^{i2\pi\varphi_{\zeta_A}(\Phi)}, \eta_A(\Phi)e^{i2\pi\varphi_{\eta_A}(\Phi)}\right)$.

Definition 7 Liu et al. (2020a) Let $A = \left(\zeta_A(\Phi)e^{i2\pi\varphi_{\zeta_A}(\Phi)}, \eta_A(\Phi)e^{i2\pi\varphi_{\eta_A}(\Phi)}\right)$ and $B = \left(\zeta_B(\Phi)e^{i2\pi\varphi_{\zeta_B}(\Phi)}, \eta_B(\Phi)e^{i2\pi\varphi_{\eta_B}(\Phi)}\right)$ are two Cq-ROFNs. Then,

1. $A \subseteq B$ iff $\zeta_A(\Phi) \le \zeta_B(\Phi), \varphi_{\zeta_A}(\Phi) \le \varphi_{\zeta_B}(\Phi)$ and $\eta_A(\Phi) \ge \eta_B(\Phi), \varphi_{\eta_A}(\Phi) \ge \varphi_{\eta_B}(\Phi)$;
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
3. $A^c = \left(\eta_A(\Phi)e^{i2\pi\varphi_{\eta_A}(\Phi)}, \zeta_A(\Phi)e^{i2\pi\varphi_{\zeta_A}(\Phi)}\right)$.

3 Generalized VCSMs for Cq-ROFSs

To investigate the closeness between any two numbers, we needed several sorts of measures for determining the closeness among any two objects. Under the above circumstances, we try to employ several sorts of measures, called four types of VCSMs and generalized VCSMs for the Cq-ROFS. The proposed approaches are given as follows.

3.1 The VCSM for Cq-ROFSs

In the presence of the parameter and their influence, we will determine a lot of measures in shape: four types of VCSMs and verify their properties. The special cases of the proposed methods are also discussed. Then, we examine the distance measures for the Cq-ROFSs.

Definition 8 The HVSM for a Cq-ROFS is given by:

$$H_V^1(A,B) = \frac{1}{n}\left(\begin{array}{l} \frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right) + \left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)} \\ + \frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i)\right) + \left(\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i)\right)} \end{array}\right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)}} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i)}} \end{array}\right) \end{array}\right) \quad (7)$$

Equation (7) holds the following conditions.

1. $0 \le H_V^1(A,B) \le 1$
2. $H_V^1(A,B) = H_V^1(B,A)$
3. $H_V^1(A,B) = 1$, for $A = B$, i.e.,

$\zeta_A(\Phi_i) = \zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$

, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (7), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (7), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (7) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (7), we achieve the conception of VCSM for CIFS, when $0 \le \zeta_A^2(\Phi) + \eta_A^2(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \le 1$ in Eq. (7), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$ in Eq. (7), we achieve the conception of VCSM for Cq-ROFS.

Remark 1 The variation coefficient distance measure is denoted and defined by: $D_V^1(A,B) = 1 - H_V^1(A,B)$.

Definition 9 The HWVSM for the Cq-ROFS is given by:

$$H_{\mathrm{WV}}^{1}(A,B)=\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{8}$$

Equation (8) holds the following conditions.

1. $0\leq H_{\mathrm{WV}}^{1}(A,B)\leq 1$
2. $H_{\mathrm{WV}}^{1}(A,B)=H_{\mathrm{WV}}^{1}(B,A)$
3. $H_{\mathrm{WV}}^{1}(A,B)=1$, for $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$.

Proof The proof of Eq. (8) is given as follows:

1. It is clear that $H_{\mathrm{WV}}^{1}(A,B)\geq 0$. We considered the dice similarity measures and cosine similarity measures taken from Mahmood and Ali (2020). Then, the dice similarity measures and cosine similarity measures for q-ROFSs are given as follows:

$$0\leq\frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\leq 1$$

$$0\leq\frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\leq 1$$

$$0\leq\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\leq 1$$

and

$$0\leq\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\leq 1$$

Equation (8) follows:

$$H_{\mathrm{WV}}^{1}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}(1+1)+\\\frac{(1-\theta)}{2}(1+1)\end{array}\right)=\theta+(1-\theta)=1$$

Hence, $0\leq H_{\mathrm{WV}}^{1}(A,B)\leq 1$.

2. It is obvious that $H_{\mathrm{WV}}^{1}(A,B)=H_{\mathrm{WV}}^{1}(B,A)$.
3. $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i)$, $\varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$. Then,

$$H_{\mathrm{WV}}^{1}(A,B)=\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right)$$

$$=\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{2\left(\zeta_A^q(\Phi_i)\zeta_A^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_A^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}+\\\frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_A}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_A}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_A^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_A^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_A}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_A}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right)$$

$$= \left(\frac{\theta}{2}(1+1) + \frac{(1-\theta)}{2}(1+1)\right) = \theta + (1-\theta) = 1$$

Hence, the proof is completed.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (8), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (8), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (8) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (8), we achieve the conception of VCSM for CIFS, when $0 \le \zeta_A^2(\Phi) + \eta_A^2(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \le 1$ in Eq. (8), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$ in Eq. (8), we achieve the conception of VCSM for Cq-ROFS.

Remark 2 The weighted variation coefficient distance measure is denoted and defined by: $D_{\mathrm{WV}}^1(A,B) = 1 - H_{\mathrm{DV}}^1(A,B)$.

Definition 10 The HVSM for the Cq-ROFS is given by:

$$H_V^2(A,B) = \frac{1}{n}\left(\begin{array}{l} \frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)\right) + \left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)\right)} \\ + \frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i) + \varphi_{\xi_A}^{2q}(\Phi_i)\right) + \left(\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i) + \varphi_{\xi_B}^{2q}(\Phi_i)\right)} \end{array}\right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)}} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i) + \varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i) + \varphi_{\xi_B}^{2q}(\Phi_i)}} \end{array}\right) \end{array}\right) \quad (9)$$

Equation (9) holds the following conditions.

1. $0 \le H_V^2(A,B) \le 1$
2. $H_V^2(A,B) = H_V^2(B,A)$
3. $H_V^2(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (9), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (9), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (9) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (9), we achieve the conception of VCSM for CIFS, when $0 \le \zeta_A^2(\Phi) + \eta_A^2(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \le 1$ in Eq. (9), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$ in Eq. (9), we achieve the conception of VCSM for Cq-ROFS.

Remark 3 The variation coefficient distance measure is denoted and defined by: $D_V^2(A,B) = 1 - H_V^2(A,B)$.

Definition 11 The HWVSM for the Cq-ROFS is given by:

$$H_{\mathrm{WV}}^{2}(A,B)=\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{2\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{2\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)\right)+\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{10}$$

Equation (10) holds the following conditions.

1. $0\leq H_{\mathrm{WV}}^{2}(A,B)\leq 1$
2. $H_{\mathrm{WV}}^{2}(A,B)=H_{\mathrm{WV}}^{2}(B,A)$
3. $H_{\mathrm{WV}}^{2}(A,B)=1$, for $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i),\varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i),\varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi)=\eta_A(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (10), we achieve the conception of VCSM for FS, when $\eta_A(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (10), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (10) with a technique $0\leq\zeta_A(\Phi)+\eta_A(\Phi)\leq 1$, we achieve the conception of VCSM for IFS, when $0\leq\zeta_A(\Phi)+\eta_A(\Phi)\leq 1$ and $0\leq\varphi_{\zeta_A}(\Phi)+\varphi_{\eta_A}(\Phi)\leq 1$ in Eq. (10), we achieve the conception of VCSM for CIFS, when $0\leq\zeta_A^2(\Phi)+\eta_A^2(\Phi)\leq 1$ and $0\leq\varphi_{\zeta_A}^2(\Phi)+\varphi_{\eta_A}^2(\Phi)\leq 1$ in Eq. (10), we achieve the conception of VCSM for CPFS, and when $0\leq\zeta_A^q(\Phi)+\eta_A^q(\Phi)\leq 1$ and $0\leq\varphi_{\zeta_A}^q(\Phi)+\varphi_{\eta_A}^q(\Phi)\leq 1$ in Eq. (10), we achieve the conception of VCSM for Cq-ROFS.

Remark 4 The weighted variation coefficient distance measure is denoted and defined by: $D_{\mathrm{WV}}^{2}(A,B)=1-H_{\mathrm{WV}}^{2}(A,B)$.

Definition 12 The HVSM for the Cq-ROFS is given by:

$$H_{V}^{3}(A,B)=\frac{1}{n}\left(\begin{array}{l}\frac{\theta}{2}\left(\begin{array}{l}\frac{2\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{2\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+\sum_{i=1}^{n}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{11}$$

Equation (11) holds the following conditions.

1. $0 \le H_V^3(A,B) \le 1$
2. $H_V^3(A,B) = H_V^3(B,A)$
3. $H_V^3(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (11), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (11), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (11) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (11), we achieve the conception of VCSM for CIFS, when $0 \le \zeta_A^2(\Phi) + \eta_A^2(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \le 1$ in Eq. (11), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$ in Eq. (11), we achieve the conception of VCSM for Cq-ROFS.

Remark 5 The variation coefficient distance measure is denoted and defined by: $D_V^3(A,B) = 1 - H_V^3(A,B)$.

Definition 13 The HWVSM for the Cq-ROFS is given by:

$$H_{WV}^3(A,B) = \left(\begin{array}{l} \frac{\theta}{2}\left(\frac{2\sum_{i=1}^n \omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum_{i=1}^n \omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right) + \sum_{i=1}^n \omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)} + \frac{2\sum_{i=1}^n \omega_i{}^q\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sum_{i=1}^n \omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i)\right) + \sum_{i=1}^n \omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i)\right)} \right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^n \left(\frac{\sum_{i=1}^n \omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^n \omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^n \omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)}} + \frac{\sum_{i=1}^n \omega_i{}^q\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^n \omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^n \omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i)\right)}} \right) \end{array} \right) \quad (12)$$

Equation (12) holds the following conditions.

1. $0 \le H_{WV}^3(A,B) \le 1$
2. $H_{WV}^3(A,B) = H_{WV}^3(B,A)$
3. $H_{WV}^3(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (12), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (12), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (12) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (12), we achieve the conception of VCSM for CIFS, when $0 \le \zeta_A^2(\Phi) + \eta_A^2(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \le 1$ in Eq. (12), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta_A^q(\Phi) + \eta_A^q(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \le 1$ in Eq. (12), we achieve the conception of VCSM for Cq-ROFS.

Remark 6 The weighted variation coefficient distance measure is denoted and defined by: $D^3_{\mathrm{WV}}(A,B) = 1 - H^3_{\mathrm{WV}}(A,B)$.

Definition 14 The HVSM for the Cq-ROFS is given by:

$$H^4_V(A,B) = \frac{1}{n}\left(\begin{array}{c} \frac{\theta}{2}\left(\begin{array}{c} \frac{2\sum_{i=1}^n\left(\zeta^q_A(\Phi_i)\zeta^q_B(\Phi_i) + \eta^q_A(\Phi_i)\eta^q_B(\Phi_i) + \xi^q_A(\Phi_i)\xi^q_B(\Phi_i)\right)}{\sum_{i=1}^n\left(\zeta^{2q}_A(\Phi_i) + \eta^{2q}_A(\Phi_i) + \xi^{2q}_A(\Phi_i)\right) + \sum_{i=1}^n\left(\zeta^{2q}_B(\Phi_i) + \eta^{2q}_B(\Phi_i) + \xi^{2q}_B(\Phi_i)\right)} \\ + \frac{2\sum_{i=1}^n\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i) + \varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sum_{i=1}^n\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i) + \varphi^{2q}_{\xi_A}(\Phi_i)\right) + \sum_{i=1}^n\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i) + \varphi^{2q}_{\xi_B}(\Phi_i)\right)} \end{array}\right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^n\left(\begin{array}{c} \frac{\sum_{i=1}^n\left(\zeta^q_A(\Phi_i)\zeta^q_B(\Phi_i) + \eta^q_A(\Phi_i)\eta^q_B(\Phi_i) + \xi^q_A(\Phi_i)\xi^q_B(\Phi_i)\right)}{\sqrt{\sum_{i=1}^n\left(\zeta^{2q}_A(\Phi_i) + \eta^{2q}_A(\Phi_i) + \xi^{2q}_A(\Phi_i)\right)}.\sqrt{\sum_{i=1}^n\left(\zeta^{2q}_B(\Phi_i) + \eta^{2q}_B(\Phi_i) + \xi^{2q}_B(\Phi_i)\right)}} \\ + \frac{\sum_{i=1}^n\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i) + \varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^n\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i) + \varphi^{2q}_{\xi_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^n\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i) + \varphi^{2q}_{\xi_B}(\Phi_i)\right)}} \end{array}\right) \end{array}\right) \tag{13}$$

Equation (13) holds the following conditions.

1. $0 \le H^4_V(A,B) \le 1$
2. $H^4_V(A,B) = H^4_V(B,A)$
3. $H^4_V(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (13), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (13), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (13) with a technique $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$, we achieve the conception of VCSM for IFS, when $0 \le \zeta_A(\Phi) + \eta_A(\Phi) \le 1$ and $0 \le \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \le 1$ in Eq. (13), we achieve the conception of VCSM for CIFS, when $0 \le \zeta^2_A(\Phi) + \eta^2_A(\Phi) \le 1$ and $0 \le \varphi^2_{\zeta_A}(\Phi) + \varphi^2_{\eta_A}(\Phi) \le 1$ in Eq. (13), we achieve the conception of VCSM for CPFS, and when $0 \le \zeta^q_A(\Phi) + \eta^q_A(\Phi) \le 1$ and $0 \le \varphi^q_{\zeta_A}(\Phi) + \varphi^q_{\eta_A}(\Phi) \le 1$ in Eq. (13), we achieve the conception of VCSM for Cq-ROFS.

Remark 7 The variation coefficient distance measure is denoted and defined by: $D^4_V(A,B) = 1 - H^4_V(A,B)$.

Definition 15 The HWVSM for the Cq-ROFS is given by:

$$H_{\mathrm{WV}}^{4}(A,B)=\left(\begin{array}{l}\frac{\theta}{2}\left(\begin{array}{l}\frac{2\sum_{i=1}^{n}\omega_i{}^{q}\left(\zeta_A^{q}(\Phi_i)\zeta_B^{q}(\Phi_i)+\eta_A^{q}(\Phi_i)\eta_B^{q}(\Phi_i)+\xi_A^{q}(\Phi_i)\xi_B^{q}(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{2\sum_{i=1}^{n}\omega_i{}^{q}\left(\varphi_{\zeta_A}^{q}(\Phi_i)\varphi_{\zeta_B}^{q}(\Phi_i)+\varphi_{\eta_A}^{q}(\Phi_i)\varphi_{\eta_B}^{q}(\Phi_i)+\varphi_{\xi_A}^{q}(\Phi_i)\varphi_{\xi_B}^{q}(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)\right)+\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\omega_i{}^{q}\left(\zeta_A^{q}(\Phi_i)\zeta_B^{q}(\Phi_i)+\eta_A^{q}(\Phi_i)\eta_B^{q}(\Phi_i)+\xi_A^{q}(\Phi_i)\xi_B^{q}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\omega_i{}^{q}\left(\varphi_{\zeta_A}^{q}(\Phi_i)\varphi_{\zeta_B}^{q}(\Phi_i)+\varphi_{\eta_A}^{q}(\Phi_i)\varphi_{\eta_B}^{q}(\Phi_i)+\varphi_{\xi_A}^{q}(\Phi_i)\varphi_{\xi_B}^{q}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{14}$$

Equation (14) holds the following conditions.

1. $0\le H_{\mathrm{WV}}^{4}(A,B)\le 1$
2. $H_{\mathrm{WV}}^{4}(A,B)=H_{\mathrm{WV}}^{4}(B,A)$
3. $H_{\mathrm{WV}}^{4}(A,B)=1$, for $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$.

With the help of $\varphi_{\zeta_A}(\Phi)=\eta_A(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (14), we achieve the conception of VCSM for FS, when $\eta_A(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (14), we achieve the conception of VCSM for CFS, when $\varphi_{\zeta_A}(\Phi)=\varphi_{\eta_A}(\Phi)=0$ in Eq. (14) with a technique $0\le\zeta_A(\Phi)+\eta_A(\Phi)\le 1$, we achieve the conception of VCSM for IFS, when $0\le\zeta_A(\Phi)+\eta_A(\Phi)\le 1$ and $0\le\varphi_{\zeta_A}(\Phi)+\varphi_{\eta_A}(\Phi)\le 1$ in Eq. (14), we achieve the conception of VCSM for CIFS, when $0\le\zeta_A^{2}(\Phi)+\eta_A^{2}(\Phi)\le 1$ and $0\le\varphi_{\zeta_A}^{2}(\Phi)+\varphi_{\eta_A}^{2}(\Phi)\le 1$ in Eq. (14), we achieve the conception of VCSM for CPFS, and when $0\le\zeta_A^{q}(\Phi)+\eta_A^{q}(\Phi)\le 1$ and $0\le\varphi_{\zeta_A}^{q}(\Phi)+\varphi_{\eta_A}^{q}(\Phi)\le 1$ in Eq. (14), we achieve the conception of VCSM for Cq-ROFS.

Remark 8 The weighted variation coefficient distance measure is denoted and defined by: $D_{\mathrm{WV}}^{4}(A,B)=1-H_{\mathrm{WV}}^{4}(A,B)$.

3.2 The generalized VCSM for Cq-ROFSs

In the presence of the parameter and their influence, we will determine a lot of measures in shape: four types of generalized VCSMs and verify their properties. The special cases of the proposed methods are also discussed. Then, we propose the generalized variation coefficient distance measures for Cq-ROFSs.

Definition 16 The GHVSM for the Cq-ROFS is given by:

$$H^1_{GV}(A,B)=\frac{1}{n}\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sigma\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+(1-\sigma)\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sigma\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)\right)+(1-\sigma)\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sqrt{\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)}.\sqrt{\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)}}\end{array}\right)\end{array}\right) \quad (15)$$

Equation (15) holds the following conditions.

1. $0 \le H^1_{GV}(A,B) \le 1$
2. $H^1_{GV}(A,B) = H^1_{GV}(B,A)$
3. $H^1_{GV}(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Remark 9 The variation coefficient distance measure is denoted and defined by: $D^1_{GV}(A,B) = 1 - H^1_{GV}(A,B)$. If we consider $\sigma = 0.5$, then Eq. (15) is reduced to Eq. (8). By changing the value of the parameter $\sigma = 0$, then Eq. (15) is converted into asymmetric or projection measures as follows:

$$H^1_{GV}(A,B)=\frac{1}{n}\left(\begin{array}{l}\frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sqrt{\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)}.\sqrt{\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)}}\end{array}\right)\end{array}\right) \quad (16)$$

By changing the value of the parameter $\sigma = 1$, then Eq. (15) is converted into asymmetric or projection measures as follows:

$$H^{1}_{\mathrm{GV}}(A,B)=\frac{1}{n}\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{17}$$

Definition 17 The GHWVSM for the Cq-ROFS is given by:

$$H^{1}_{\mathrm{GWV}}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sigma\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+(1-\sigma)\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sigma\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+(1-\sigma)\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{18}$$

Equation (18) holds the following conditions.

1. $0 \le H^{1}_{\mathrm{GWV}}(A,B) \le 1$
2. $H^{1}_{\mathrm{GWV}}(A,B) = H^{1}_{\mathrm{GWV}}(B,A)$
3. $H^{1}_{\mathrm{GWV}}(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i), \varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Remark 10 The weighted variation coefficient distance measure is denoted and defined by: $D^{1}_{\mathrm{GWV}}(A,B) = 1 - H^{1}_{\mathrm{GDV}}(A,B)$. If we consider $\sigma = 0.5$, then Eq. (18) is reduced to Eq. (9). By changing the value of the parameter $\sigma = 0$, then Eq. (18) is converted into asymmetric or projection measures as follows:

$$H^{1}_{\mathrm{GWV}}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right)\tag{19}$$

By changing the value of the parameter $\sigma=1$, then Eq. (18) is converted into asymmetric or projection measures as follows:

$$H^{1}_{\mathrm{GWV}}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right)\tag{20}$$

Definition 18 The GHVSM for the Cq-ROFS is given by:

$$H^{2}_{\mathrm{GV}}(A,B)=\frac{1}{n}\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sigma\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+(1-\sigma)\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sigma\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)\right)+(1-\sigma)\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right)\tag{21}$$

Equation (21) holds the following condition.

1. $0 \le H_{\mathrm{GV}}^2(A,B) \le 1$
2. $H_{\mathrm{GV}}^2(A,B) = H_{\mathrm{GV}}^2(B,A)$
3. $H_{\mathrm{GV}}^2(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Remark 11 The variation coefficient distance measure is denoted and defined by: $D_{\mathrm{GV}}^2(A,B) = 1 - H_{\mathrm{GV}}^2(A,B)$. If we consider $\sigma = 0.5$, then Eq. (21) is reduced to Eq. (10). By changing the value of the parameter $\sigma = 0$, then Eq. (21) is converted into asymmetric or projection measures as follows:

$$H_{\mathrm{GV}}^2(A,B) = \frac{1}{n}\left(\begin{array}{l} \frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)\right)} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i) + \varphi_{\xi_B}^{2q}(\Phi_i)\right)} \end{array}\right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)}} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i) + \varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i) + \varphi_{\xi_B}^{2q}(\Phi_i)}} \end{array}\right) \end{array}\right) \tag{22}$$

By changing the value of the parameter $\sigma = 1$, then Eq. (21) is converted into asymmetric or projection measures as follows:

$$H_{\mathrm{GV}}^2(A,B) = \frac{1}{n}\left(\begin{array}{l} \frac{\theta}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)\right)} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i) + \varphi_{\xi_A}^{2q}(\Phi_i)\right)} \end{array}\right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)}} \\ + \frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i) + \varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i) + \varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i) + \varphi_{\eta_A}^{2q}(\Phi_i) + \varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i) + \varphi_{\eta_B}^{2q}(\Phi_i) + \varphi_{\xi_B}^{2q}(\Phi_i)}} \end{array}\right) \end{array}\right) \tag{23}$$

Definition 19 The GHWVSM for the Cq-ROFS is given by:

Equation (24) holds the following conditions.

$$H_{\mathrm{GWV}}^{2}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sigma\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+(1-\sigma)\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sigma\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)\right)+(1-\sigma)\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{24}$$

1. $0\leq H_{\mathrm{GWV}}^{2}(A,B)\leq 1$
2. $H_{\mathrm{GWV}}^{2}(A,B)=H_{\mathrm{GWV}}^{2}(B,A)$
3. $H_{\mathrm{GWV}}^{2}(A,B)=1$, for $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$.

Remark 12 The weighted variation coefficient distance measure is denoted and defined by: $D_{\mathrm{GWV}}^{2}(A,B)=1-H_{\mathrm{GWV}}^{2}(A,B)$. If we consider $\sigma=0.5$, then Eq. (24) is reduced to Eq. (11). By changing the value of the parameter $\sigma=0$, then Eq. (24) is converted into asymmetric or projection measures as follows:

$$H_{\mathrm{GWV}}^{2}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i\left(\begin{array}{c}\frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)}}\\+\frac{\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)+\varphi_{\xi_A}^q(\Phi_i)\varphi_{\xi_B}^q(\Phi_i)\right)}{\sqrt{\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)+\varphi_{\xi_A}^{2q}(\Phi_i)}.\sqrt{\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)+\varphi_{\xi_B}^{2q}(\Phi_i)}}\end{array}\right)\end{array}\right) \tag{25}$$

By changing the value of the parameter $\sigma = 1$, then Eq. (24) is converted into asymmetric or projection measures as follows:

$$H^{2}_{\mathrm{GWV}}(A,B) = \left(\begin{array}{l} \frac{\theta}{2}\sum_{i=1}^{n}\omega_i \left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)\right)} \\ + \frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i) + \varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i) + \varphi^{2q}_{\xi_A}(\Phi_i)\right)} \end{array} \right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\omega_i \left(\begin{array}{l} \frac{\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i) + \xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i) + \xi_A^{2q}(\Phi_i)}.\sqrt{\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i) + \xi_B^{2q}(\Phi_i)}} \\ + \frac{\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i) + \varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i) + \varphi^{2q}_{\xi_A}(\Phi_i)}.\sqrt{\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i) + \varphi^{2q}_{\xi_B}(\Phi_i)}} \end{array} \right) \end{array} \right) \tag{26}$$

Definition 20 The GHVSM for the Cq-ROFS is given by:

$$H^{3}_{\mathrm{GV}}(A,B) = \frac{1}{n}\left(\begin{array}{l} \frac{\theta}{2}\left(\begin{array}{l} \frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right) + (1-\sigma)\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)} \\ + \frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i)\right) + (1-\sigma)\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i)\right)} \end{array} \right) + \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l} \frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)}} \\ \frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i)\right)}} \end{array} \right) \end{array} \right) \tag{27}$$

Equation (27) holds the following conditions.

1. $0 \le H^3_{\mathrm{GV}}(A,B) \le 1$
2. $H^3_{\mathrm{GV}}(A,B) = H^3_{\mathrm{GV}}(B,A)$
3. $H^3_{\mathrm{GV}}(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Remark 13 The variation coefficient distance measure is denoted and defined by: $D^3_{\mathrm{GV}}(A,B) = 1 - H^3_{\mathrm{GV}}(A,B)$. If we consider $\sigma = 0.5$, then Eq. (27) is reduced to Eq. (12). By changing the value of the parameter $\sigma = 0$, then Eq. (27) is converted into asymmetric or projection measures as follows:

Equation (30) holds the following conditions.

$$H_{GV}^{3}(A,B)=\frac{1}{n}\left(\begin{array}{c}\frac{\theta}{2}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{28}$$

By changing the value of the parameter $\sigma = 1$, then Eq. (27) is converted into asymmetric or projection measures as follows:

$$H_{GV}^{3}(A,B)=\frac{1}{n}\left(\begin{array}{c}\frac{\theta}{2}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{29}$$

Definition 21 The GHWVSM for the Cq-ROFS is given by:

$$H_{GWV}^{3}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi_{\zeta_A}^q(\Phi_i)\varphi_{\zeta_B}^q(\Phi_i)+\varphi_{\eta_A}^q(\Phi_i)\varphi_{\eta_B}^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_A}^{2q}(\Phi_i)+\varphi_{\eta_A}^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi_{\zeta_B}^{2q}(\Phi_i)+\varphi_{\eta_B}^{2q}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{30}$$

1. $0 \leq H^3_{\mathrm{GWV}}(A,B) \leq 1$
2. $H^3_{\mathrm{GWV}}(A,B) = H^3_{\mathrm{GWV}}(B,A)$
3. $H^3_{\mathrm{GWV}}(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Remark 14 The weighted variation coefficient distance measure is denoted and defined by: $D^3_{\mathrm{GWV}}(A,B) = 1 - H^3_{\mathrm{GWV}}(A,B)$. If we consider $\sigma = 0.5$, then Eq. (30) is reduced to Eq. (13). By changing the value of the parameter $\sigma = 0$, then Eq. (30) is converted into asymmetric or projection measures as follows:

$$H^3_{\mathrm{GWV}}(A,B) = \left(\begin{array}{c} \frac{\theta}{2}\left(\begin{array}{c} \frac{\sum_{i=1}^{n} \omega_i{}^q \left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)} \\ + \frac{\sum_{i=1}^{n} \omega_i{}^q \left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i)\right)} \end{array} \right) \\ + \frac{(1-\theta)}{2}\sum_{i=1}^{n} \left(\begin{array}{c} \frac{\sum_{i=1}^{n} \omega_i{}^q \left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)}} \\ + \frac{\sum_{i=1}^{n} \omega_i{}^q \left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i)\right)}} \end{array} \right) \end{array} \right) \quad (31)$$

By changing the value of the parameter $\sigma = 1$, then Eq. (30) is converted into asymmetric or projection measures as follows:

$$H^3_{\mathrm{GWV}}(A,B) = \left(\begin{array}{c} \frac{\theta}{2}\left(\begin{array}{c} \frac{\sum\limits_{i=1}^{n} \omega_i{}^q \left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right)} \\ + \frac{\sum\limits_{i=1}^{n} \omega_i{}^q \left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i)\right)} \end{array} \right) \\ + \frac{(1-\theta)}{2}\sum\limits_{i=1}^{n} \left(\begin{array}{c} \frac{\sum\limits_{i=1}^{n} \omega_i{}^q \left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i) + \eta_A^q(\Phi_i)\eta_B^q(\Phi_i)\right)}{\sqrt{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i) + \eta_A^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i) + \eta_B^{2q}(\Phi_i)\right)}} \\ + \frac{\sum\limits_{i=1}^{n} \omega_i{}^q \left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i) + \varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)\right)}{\sqrt{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i) + \varphi^{2q}_{\eta_A}(\Phi_i)\right)}\cdot\sqrt{\sum\limits_{i=1}^{n} \omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i) + \varphi^{2q}_{\eta_B}(\Phi_i)\right)}} \end{array} \right) \end{array} \right) \quad (32)$$

Definition 22 The GHVSM for the Cq-ROFS is given by:

Equation (33) holds the following conditions.

$$H^4_{GV}(A,B)=\frac{1}{n}\left(\begin{array}{l}\frac{\theta}{2}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{33}$$

1. $0\leq H^4_{GV}(A,B)\leq 1$
2. $H^4_{GV}(A,B)=H^4_{GV}(B,A)$
3. $H^4_{GV}(A,B)=1$, for $A=B$, i.e., $\zeta_A(\Phi_i)=\zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i)=\varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i)=\varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i)=\eta_B(\Phi_i)$.

Remark 15 The variation coefficient distance measure is denoted and defined by: $D^4_{GV}(A,B)=1-H^4_{GV}(A,B)$. If we consider $\sigma=0.5$, then Eq. (33) is reduced to Eq. (14). By changing the value of the parameter $\sigma=0$, then Eq. (33) is converted into asymmetric or projection measures as follows:

$$H^4_{GV}(A,B)=\frac{1}{n}\left(\begin{array}{l}\frac{\theta}{2}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{34}$$

By changing the value of the parameter $\sigma = 1$, then Eq. (33) is converted into asymmetric or projection measures as follows:

$$H^4_{\mathrm{GV}}(A,B) = \frac{1}{n}\left(\begin{array}{c} \frac{\theta}{2}\left(\begin{array}{c} \frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)} \\ +\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)} \end{array}\right) \\ +\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c} \frac{\sum_{i=1}^{n}\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}} \\ +\frac{\sum_{i=1}^{n}\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}} \end{array}\right) \end{array}\right) \tag{35}$$

Definition 23 The GHWVSM for the Cq-ROFS is given by:

$$H^4_{\mathrm{GWV}}(A,B) = \left(\begin{array}{c} \frac{\theta}{2}\left(\begin{array}{c} \frac{\sum_{i=1}^{n}{\omega_i}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}{\omega_i}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}{\omega_i}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)} \\ +\frac{\sum_{i=1}^{n}{\omega_i}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sigma\sum_{i=1}^{n}{\omega_i}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)+(1-\sigma)\sum_{i=1}^{n}{\omega_i}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)} \end{array}\right) \\ +\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c} \frac{\sum_{i=1}^{n}{\omega_i}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}{\omega_i}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}{\omega_i}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}} \\ +\frac{\sum_{i=1}^{n}{\omega_i}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}{\omega_i}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}\cdot\sqrt{\sum_{i=1}^{n}{\omega_i}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}} \end{array}\right) \end{array}\right) \tag{36}$$

Fig. 2 Expression of the carcinoma in the human brain

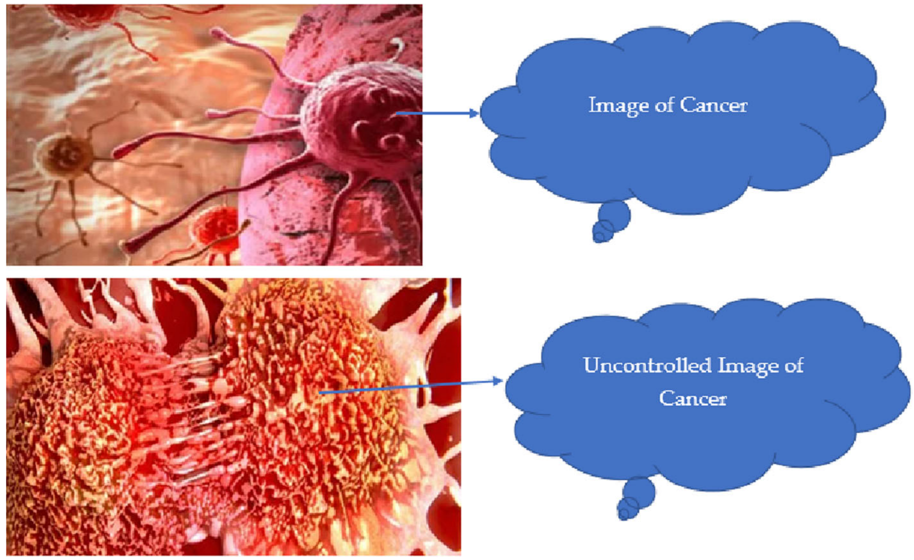

Table 1 Complex intuitionistic fuzzy decision matrix

Symbols	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix}0.6e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.2)}\end{pmatrix}$	$\begin{pmatrix}0.9e^{i2\pi(0.8)}, \\ 0.1e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.5e^{i2\pi(0.4)}, \\ 0.3e^{i2\pi(0.4)}\end{pmatrix}$	$\begin{pmatrix}0.6e^{i2\pi(0.4)}, \\ 0.2e^{i2\pi(0.1)}\end{pmatrix}$
A_2	$\begin{pmatrix}0.4e^{i2\pi(0.2)}, \\ 0.3e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.5e^{i2\pi(0.3)}, \\ 0.1e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.6e^{i2\pi(0.4)}, \\ 0.2e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.8e^{i2\pi(0.6)}, \\ 0.1e^{i2\pi(0.2)}\end{pmatrix}$
A_3	$\begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.2)}\end{pmatrix}$	$\begin{pmatrix}0.4e^{i2\pi(0.6)}, \\ 0.3e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.1e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.6e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.4)}\end{pmatrix}$
A_4	$\begin{pmatrix}0.7e^{i2\pi(0.6)}, \\ 0.3e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.4e^{i2\pi(0.9)}, \\ 0.2e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.7e^{i2\pi(0.7)}, \\ 0.2e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.5e^{i2\pi(0.3)}, \\ 0.3e^{i2\pi(0.6)}\end{pmatrix}$
A_5	$\begin{pmatrix}0.2e^{i2\pi(0.8)}, \\ 0.5e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.7e^{i2\pi(0.3)}, \\ 0.3e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.6e^{i2\pi(0.5)}, \\ 0.1e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.6e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.4)}\end{pmatrix}$

Table 2 Complex intuitionistic fuzzy information of reference set

Symbols	C_1	C_2	C_3	C_4
B	$\begin{pmatrix}0.7e^{i2\pi(0.5)}, \\ 0.1e^{i2\pi(0.3)}\end{pmatrix}$	$\begin{pmatrix}0.4e^{i2\pi(0.6)}, \\ 0.5e^{i2\pi(0.2)}\end{pmatrix}$	$\begin{pmatrix}0.5e^{i2\pi(0.5)}, \\ 0.3e^{i2\pi(0.1)}\end{pmatrix}$	$\begin{pmatrix}0.8e^{i2\pi(0.7)}, \\ 0.2e^{i2\pi(0.1)}\end{pmatrix}$

Equation (36) holds the following conditions.

1. $0 \leq H^4_{\text{GWV}}(A,B) \leq 1$
2. $H^4_{\text{GWV}}(A,B) = H^4_{\text{GWV}}(B,A)$
3. $H^4_{\text{GWV}}(A,B) = 1$, for $A = B$, i.e., $\zeta_A(\Phi_i) = \zeta_B(\Phi_i)$, $\varphi_{\zeta_A}(\Phi_i) = \varphi_{\zeta_B}(\Phi_i), \varphi_{\eta_A}(\Phi_i) = \varphi_{\eta_B}(\Phi_i)$, and $\eta_A(\Phi_i) = \eta_B(\Phi_i)$.

Table 3 The required value is obtained from complex intuitionistic fuzzy numbers

Methods	(A_1,B)	(A_2,B)	(A_3,B)	(A_4,B)	(A_5,B)
H^4_V	0.513	0.5094	0.5093	0.514	0.5095
H^4_{WV}	0.135	0.133	0.1343	0.136	0.1335

Remark 16 The weighted variation coefficient distance measure is denoted and defined by: $D^4_{\text{GWV}}(A,B) = 1 - H^4_{\text{GWV}}(A,B)$. If we consider $\sigma = 0.5$, then Eq. (36) is reduced to Eq. (15). By changing the value of the parameter $\sigma = 0$, then Eq. (36) is converted into asymmetric or projection measures as follows:

$$H^4_{\mathrm{GWV}}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{37}$$

By changing the value of the parameter $\sigma = 1$, then Eq. (36) is converted into asymmetric or projection measures as follows:

With the help of $\varphi_{\zeta_A}(\Phi) = \eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (15)–Eq. (38), we achieve the conception of VCSM for FS, when $\eta_A(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (15)–Eq. (38), we achieve the conception of VCSM for CFS, when

$$H^4_{\mathrm{GWV}}(A,B)=\left(\begin{array}{c}\frac{\theta}{2}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}\end{array}\right)\\+\frac{(1-\theta)}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\zeta_A^q(\Phi_i)\zeta_B^q(\Phi_i)+\eta_A^q(\Phi_i)\eta_B^q(\Phi_i)+\xi_A^q(\Phi_i)\xi_B^q(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_A^{2q}(\Phi_i)+\eta_A^{2q}(\Phi_i)+\xi_A^{2q}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\zeta_B^{2q}(\Phi_i)+\eta_B^{2q}(\Phi_i)+\xi_B^{2q}(\Phi_i)\right)}}\\+\frac{\sum_{i=1}^{n}\omega_i{}^q\left(\varphi^q_{\zeta_A}(\Phi_i)\varphi^q_{\zeta_B}(\Phi_i)+\varphi^q_{\eta_A}(\Phi_i)\varphi^q_{\eta_B}(\Phi_i)+\varphi^q_{\xi_A}(\Phi_i)\varphi^q_{\xi_B}(\Phi_i)\right)}{\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_A}(\Phi_i)+\varphi^{2q}_{\eta_A}(\Phi_i)+\varphi^{2q}_{\xi_A}(\Phi_i)\right)}.\sqrt{\sum_{i=1}^{n}\omega_i{}^{2q}\left(\varphi^{2q}_{\zeta_B}(\Phi_i)+\varphi^{2q}_{\eta_B}(\Phi_i)+\varphi^{2q}_{\xi_B}(\Phi_i)\right)}}\end{array}\right)\end{array}\right) \tag{38}$$

Table 4 Ranking values are obtained from Table 3

Methods	Ranking order of the regions
H_V^1	$A_4 \geq A_1 \geq A_5 \geq A_2 \geq A_3$
H_{WV}^2	$A_4 \geq A_1 \geq A_3 \geq A_5 \geq A_2$

$\varphi_{\zeta_A}(\Phi) = \varphi_{\eta_A}(\Phi) = 0$ in Eq. (15)–Eq. (38) with a technique $0 \leq \zeta_A(\Phi) + \eta_A(\Phi) \leq 1$, we achieve the conception of VCSM for IFS, when $0 \leq \zeta_A(\Phi) + \eta_A(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}(\Phi) + \varphi_{\eta_A}(\Phi) \leq 1$ in Eq. (15)–Eq. (38), we achieve the conception of VCSM for CIFS, when $0 \leq \zeta_A^2(\Phi) + \eta_A^2(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}^2(\Phi) + \varphi_{\eta_A}^2(\Phi) \leq 1$ in Eq. (15)–Eq. (38), we achieve the conception of VCSM for CPFS, and when $0 \leq \zeta_A^q(\Phi) + \eta_A^q(\Phi) \leq 1$ and $0 \leq \varphi_{\zeta_A}^q(\Phi) + \varphi_{\eta_A}^q(\Phi) \leq 1$ in Eq. (15)–Eq. (38), we achieve the conception of VCSM for Cq-ROFS.

4 Analysis of brain carcinoma based on presented measures

Brain cancer is a rare part of cancer that starts in the tissues of the brain or spinal cord. The major symptom of cancer is unknown, but the risk factor of cancer is very complicated, and it depends on the age and health of the patient. The ratio of brain cancer in human beings is less than or equal to 2.6% in women, and 1.3% in man are occurred. Be that as it may, these are not viewed as obvious mind diseases. While the reason for cerebrum malignancy is ineffectively perceived, acquired, and ecological components are accepted to be significant in its turn of events.

4.1 Decision-making strategy

With the help of decision-making strategy, several utilizations occurred by using some prevailing theories. Nowadays, many people have been affected by brain cancer, and by using different types of sources doctors easily determine the place of the brain cancer and their weight location. This analysis tries to determine the presence of Eqs. (33) and (36), the most dangerous sort of brain cancer with the help of the invented measures for Cq-ROFNs. The invented decision-making strategy includes some procedure, for this, they have needed the family of alternatives, stated in the form of diseases $A = \{A_1, A_2, \ldots, A_n\}$ and its attributes, and expressed in the shape of symptoms $G = \{G_1, G_2, \ldots, G_m\}$. For this, we consider the term $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}^T$, where $\sum_{i=1}^{n} \omega_i = 1, \omega_i \in [0,1]$, stated the weight vector. Let us

Table 5 Complex Pythagorean fuzzy information about known materials

	A_1	A_2	A_3	A_4
Φ_1	$\begin{pmatrix} 0.8e^{i.2\pi(0.7)}, \\ 0.3e^{i.2\pi(0.4)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i.2\pi(0.5)}, \\ 0.4e^{i.2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i.2\pi(0.6)}, \\ 0.7e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i.2\pi(0.6)}, \\ 0.3e^{i.2\pi(0.5)} \end{pmatrix}$
Φ_2	$\begin{pmatrix} 0.7e^{i.2\pi(0.6)}, \\ 0.5e^{i.2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i.2\pi(0.7)}, \\ 0.6e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i.2\pi(0.5)}, \\ 0.5e^{i.2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i.2\pi(0.5)}, \\ 0.4e^{i.2\pi(0.6)} \end{pmatrix}$
Φ_3	$\begin{pmatrix} 0.9e^{i.2\pi(0.4)}, \\ 0.2e^{i.2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i.2\pi(0.8)}, \\ 0.3e^{i.2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.3e^{i.2\pi(0.6)}, \\ 0.3e^{i.2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i.2\pi(0.6)}, \\ 0.5e^{i.2\pi(0.3)} \end{pmatrix}$
Φ_4	$\begin{pmatrix} 0.6e^{i.2\pi(0.6)}, \\ 0.5e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i.2\pi(0.9)}, \\ 0.3e^{i.2\pi(0.1)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i.2\pi(0.7)}, \\ 0.5e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{i.2\pi(0.8)}, \\ 0.7e^{i.2\pi(0.2)} \end{pmatrix}$
Φ_5	$\begin{pmatrix} 0.5e^{i.2\pi(0.3)}, \\ 0.6e^{i.2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i.2\pi(0.3)}, \\ 0.5e^{i.2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i.2\pi(0.7)}, \\ 0.6e^{i.2\pi(0.3)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i.2\pi(0.6)}, \\ 0.5e^{i.2\pi(0.2)} \end{pmatrix}$
Φ_6	$\begin{pmatrix} 0.4e^{i.2\pi(0.6)}, \\ 0.7e^{i.2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.2e^{i.2\pi(0.7)}, \\ 0.8e^{i.2\pi(0.2)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i.2\pi(0.6)}, \\ 0.3e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i.2\pi(0.8)}, \\ 0.5e^{i.2\pi(0.3)} \end{pmatrix}$
Φ_7	$\begin{pmatrix} 0.2e^{i.2\pi(0.2)}, \\ 0.5e^{i.2\pi(0.8)} \end{pmatrix}$	$\begin{pmatrix} 0.1e^{i.2\pi(0.6)}, \\ 0.9e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.2e^{i.2\pi(0.5)}, \\ 0.8e^{i.2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i.2\pi(0.7)}, \\ 0.3e^{i.2\pi(0.4)} \end{pmatrix}$

Table 6 Complex Pythagorean fuzzy information about unknown materials

Symbols	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7
B	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$	$(1e^{i2\pi}, 0)$

Table 7 The required value is obtained from complex Pythagorean fuzzy numbers

Methods	(A_1, B)	(A_2, B)	(A_3, B)	(A_4, B)
H_V^4	0.501	0.563	0.572	0.564
H_{WV}^4	0.02127	0.02129	0.0225	0.0223

Table 8 The ranking value is obtained from Table 7

Methods	Ranking
H_V^1	$A_3 \geq A_4 \geq A_2 \geq A_1$
H_{WV}^2	$A_3 \geq A_4 \geq A_2 \geq A_1$

The best alternative is A_3

consider, a family of alternatives $A = \{A_1, A_2, A_3, A_4, A_5\}$ concerning alternative also called reference set B is the form of Cq-ROFNs discussed in this section. Figure 2 can help the reader how can find brain cancer and what is the real image of brain cancer.

In the above circumstance, we described some specific procedures on how we find the most dangerous part of brain cancer in human beings.

4.2 Decision-making processes

The major stages, involved in decision-making processes, are illustrated here:

Stage 1: Collect the hypothetical data in the form of Cq-ROFNs and put it into the matrix which includes rows and columns.

Stage 2: To invent the closeness between any two Cq-ROFNs, we use Eqs. (33) and (36).

Stage 3: Rank all alternatives and demonstrate the best one.

Stage 4: The end.

In the presence of the above procedure, we try to diagnose some examples for verifying the invented measures.

4.3 Numerical illustrations

To investigate the most dangerous and unsafe part of brain cancer in human beings. For this, we needed to provide a lot of data related to brain cancer. In the most essential terms, malignancy alludes to cells that outgrow control and attack different tissues. Cells might become malignant because of the amassing of imperfections, or transformations, in their DNA. Certain acquired hereditary imperfections (for instance, BRCA1 and BRCA2 transformations) and diseases can expand the danger of malignant growth. Natural components (for instance, air contamination) and helpless way of life decisions, for example, smoking and hefty liquor use—can likewise harm DNA and lead to disease. Brain cancer has more than 120 sorts, but the most dangerous five sorts of brain cancer which show the family of alternatives (unknown) are discussed below.

1. Astrocytoma’s
2. Glioblastoma multiforme
3. Meningioma
4. Craniopharyngiomas
5. Germ cell tumors

For this, we consider their attributes in the form of symptoms whose details dare be discussed below.

1. Blurred vision
2. Changes in speech
3. Confusion
4. Difficulty walking

To simplify the above problems, we use the above-invented decision-making procedure to determine the best

Table 9 Complex q-rung orthopair fuzzy information

Symbols	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix} 0.6e^{i2\pi(0.7)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.9e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i2\pi(0.4)}, \\ 0.8e^{i2\pi(0.9)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.4)}, \\ 0.7e^{i2\pi(0.6)} \end{pmatrix}$
A_2	$\begin{pmatrix} 0.4e^{i2\pi(0.2)}, \\ 0.9e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i2\pi(0.3)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.4)}, \\ 0.8e^{i2\pi(0.9)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.8)} \end{pmatrix}$
A_3	$\begin{pmatrix} 0.7e^{i2\pi(0.7)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{i2\pi(0.6)}, \\ 0.8e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i2\pi(0.7)}, \\ 0.6e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.5)}, \\ 0.8e^{i2\pi(0.9)} \end{pmatrix}$
A_4	$\begin{pmatrix} 0.7e^{i2\pi(0.6)}, \\ 0.6e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{i2\pi(0.9)}, \\ 0.5e^{i2\pi(0.4)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i2\pi(0.7)}, \\ 0.5e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i2\pi(0.3)}, \\ 0.6e^{i2\pi(0.9)} \end{pmatrix}$
A_5	$\begin{pmatrix} 0.2e^{i2\pi(0.8)}, \\ 0.9e^{i2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.7e^{i2\pi(0.3)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.5)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.6e^{i2\pi(0.5)}, \\ 0.7e^{i2\pi(0.8)} \end{pmatrix}$

Table 10 Complex q-rung orthopair fuzzy information for reference set

Symbols	C_1	C_2	C_3	C_4
B	$\begin{pmatrix} 0.7e^{i2\pi(0.5)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}$	$\begin{pmatrix} 0.4e^{i2\pi(0.6)}, \\ 0.9e^{i2\pi(0.6)} \end{pmatrix}$	$\begin{pmatrix} 0.5e^{i2\pi(0.5)}, \\ 0.7e^{i2\pi(0.5)} \end{pmatrix}$	$\begin{pmatrix} 0.8e^{i2\pi(0.7)}, \\ 0.6e^{i2\pi(0.5)} \end{pmatrix}$

Table 11 The required value obtained complex q-rung orthopair fuzzy numbers

Methods	(A_1, B)	(A_2, B)	(A_3, B)	(A_4, B)	(A_5, B)
H_V^4	0.8696	0.8690	0.86841	0.86843	0.8657
H_{WV}^4	0.2367	0.2361	0.2366	0.2365	0.2360

Table 12 A ranking value was obtained from Table 11

Methods	Ranking
H_V^1	$A_1 \geq A_2 \geq A_4 \geq A_3 \geq A_5$
H_{WV}^2	$A_1 \geq A_3 \geq A_4 \geq A_2 \geq A_5$

The best alternative is A_1

ways. The major stages, involved in decision-making processes, are illustrated here:

Stage 1: Collect the hypothetical data in the form of Cq-ROFNs and put it into the matrix which includes rows and columns, stated in the form of Table 1.

For this, we choose the decision matrix in the known alternatives in the shape of Table 2.

Stage 2: To invent the closeness between any two Cq-ROFNs, we use Eqs. (33) and (36). For this, we use the weight vector $\omega = (0.3, 0.3, 0.3, 0.1), \sigma = \theta = 0.5$, where the final values are described in Table 3.

Stage 3: Rank all alternatives and demonstrate the best one. The final ranking order of the alternatives represents the set of regions discussed in the shape of Table 4.

The best alternative to the MAGDM is A_4 for Eqs. (33) and (36). From the above analysis, the most dangerous sort of cancer is craniopharyngiomas.

Further, we diagnose some data from Ref. (Alkouri and Salleh 2012). Let us consider three known patterns $A = \{A_1, A_2, A_3, A_4, A_5\}$ concerning unknown pattern B is the form of Cq-ROFNs. The four CPFNs $A_i (i = 1, 2, 3, 4)$ with an attribute set denoted by $X = \{\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6, \Phi_7\}$, and the weight vector of attributes is denoted by $\omega = (0.11, 0.14, 0.1, 0.18, 0.21, 0.10, 0.16)^T$. The information about known and unknown building materials is in the form of CPFNs. The major stages, involved in decision-making processes, are illustrated here:

Stage 1: Collect the hypothetical data in the form of Cq-ROFNs and put it into the matrix which includes rows and columns, stated in the form of Table 5.

And the decision matrix for unknown patterns is discussed in the shape of Table 6.

Stage 2: To invent the closeness between any two Cq-ROFNs, we use Eqs. (33) and (36). For this, we use Eqs. (33) and (36), where the final values are described in Table 7.

Stage 3: Rank all alternatives and demonstrate the best one. The final ranking order for alternatives is in the shape of Table 8.

Further, based on Eqs. (33) and (36), we will propose the model of the MAGDM method with Cq-ROFNs. Let $A = \{A_1, A_2, \ldots, A_n\}$ represents the family of alternatives and $G = \{G_1, G_2, \ldots, G_m\}$ denotes the family of attributes. The weight vectors for attributes are represented by

Table 13 Comparison between proposed methods and existing methods

Concepts	Methods	Ranking
Garg and Rani (2019a)	$H_{WV}^4(A_1, B) = 0.024, H_{WV}^4(A_2, B) = 0.017, H_{WV}^4(A_3, B) = 0.026,$ $H_{WV}^4(A_4, B) = 0.029, H_{WV}^4(A_5, B) = 0.017$	$A_4 \geq A_3 \geq A_1 \geq A_5 \geq A_2$
Rani and Garg (2022)	$H_{WV}^4(A_1, B) = 0.1045, H_{WV}^4(A_2, B) = 0.0915, H_{WV}^4(A_3, B) = 0.110,$ $H_{WV}^4(A_4, B) = 0.111, H_{WV}^4(A_5, B) = 0.092$	$A_4 \geq A_3 \geq A_1 \geq A_5 \geq A_2$
Ullah et al. (2020a)	$H_{WV}^4(A_1, B) = 0.114, H_{WV}^4(A_2, B) = 0.092, H_{WV}^4(A_3, B) = 0.110,$ $H_{WV}^4(A_4, B) = 0.119, H_{WV}^4(A_5, B) = 0.099$	$A_4 \geq A_1 \geq A_3 \geq A_5 \geq A_2$
Cq-ROFS	$H_{WV}^4(A_1, B) = 0.135, H_{WV}^4(A_2, B) = 0.133, H_{WV}^4(A_3, B) = 0.1343,$ $H_{WV}^4(A_4, B) = 0.136, H_{WV}^4(A_5, B) = 0.1335$	$A_4 \geq A_1 \geq A_3 \geq A_5 \geq A_2$

Table 14 Comparison between proposed methods and existing methods

Concepts	Methods	Ranking
Garg and Rani (2019a)	Cannot be calculated	Cannot be calculated
Rani and Garg (2022)	Cannot be calculated	Cannot be calculated
Ullah et al. (2020a)	$H^4_{WV}(A_1,B)=0.0568, H^4_{WV}(A_2,B)=0.077, H^4_{WV}(A_3,B)=0.079,$ $H^4_{WV}(A_4,B)=0.067$	$A_3 \geq A_2 \geq A_4 \geq A_1$
Cq-ROFS	$H^4_{WV}(A_1,B)=0.02127, H^4_{WV}(A_2,B)=0.02129, H^4_{WV}(A_3,B)=0.0225,$ $H^4_{WV}(A_4,B)=0.0223$	$A_3 \geq A_4 \geq A_2 \geq A_1$

Table 15 Comparison between proposed methods and existing methods

Concepts	Methods	Ranking
Garg and Rani (2019a)	Cannot be calculated	Cannot be calculated
Rani and Garg (2022)	Cannot be calculated	Cannot be calculated
Ullah et al. (2020a)	Cannot be calculated	Cannot be calculated
Cq-ROFS	$H^4_{WV}(A_1,B)=0.2367, H^4_{WV}(A_2,B)=0.2361, H^4_{WV}(A_3,B)=0.2366,$ $H^4_{WV}(A_4,B)=0.2365, H^4_{WV}(A_5,B)=0.2360$	$A_1 \geq A_3 \geq A_4 \geq A_2 \geq A_5$

$\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}^T$, where $\sum_{i=1}^{n} \omega_i = 1, \omega_i \in [0,1]$. The set of alternatives is defined in Tables 1 and 2. The major stages, involved in decision-making processes, are illustrated here:

Stage 1: Collect the hypothetical data in the form of Cq-ROFNs and put it into the matrix which includes rows and columns, stated in the form of Table 9.

The unknown information is discussed in the form of Table 10.

Stage 2: To invent the closeness between any two Cq-ROFNs, we use Eqs. (33) and (36). For this, we use Eqs. (33) and (36) with weight vector $\omega = (0.3, 03, 0.3, 0.1), \sigma = \theta = 0.5$, where the final values are described in Table 11.

Stage 3: Rank all alternatives and demonstrate the best one. The final ranking order for alternatives represents in the form of Table 12.

4.4 Comparative analysis

Comparison of the invented and prevailing works is the important part of every manuscript and without comparison, the worth of any work has shown incompleted. To improve the worth of the current works, we suggested some prevailing works, proposed by Garg and Rani (2019a), Rani and Garg (2018), and Ullah et al. (2020a). In the presence of the data in Table 1, therefore, Table 13 shows the comparison of the proposed and prevailing works.

The best decision is A_4 according to the theory of Garg and Rani (2019a), Rani and Garg (2022), Ullah et al. (2020a), and proposed measures. To further find the supremacy and accuracy of the invented theory, the presence of the data in Tables 5 and 14 shows the comparison of the proposed and prevailing works.

The best decision is A_3 according to the theory of Ullah et al. (2020a) and proposed measures, but the theory of Garg and Rani (2019a) and Rani and Garg (2022) have failed, because the obtained result is computed based on the CPFSs, where the operators computed based on CIFS have not able to evaluate it because they are the special case of the CPFS. To further find the supremacy and accuracy of the invented theory, in the presence of the data in Table 9, Table 15 shows the comparison of the proposed and prevailing works.

The best decision is A_1 according to the theory of proposed measures, but the theory of Ullah et al. (2020a), Garg and Rani (2019a), and Rani and Garg (2022) has failed, because the obtained result is computed based on the Cq-ROFSs, where the operators computed based on CIFS and CIFS have not to bale to evaluate it because they are the special case of the Cq-ROFS. Furthermore, Tables 13, 14, and 15 stated that if someone gives data in the shape of Cq-ROFS, then the measures explored under CPFS, PFS, CIFS, and IFS are enabled to find their solution, similarly, if someone gives data in the shape of CPFS, PFS, CIFS, and IFS, then the measures explored under Cq-ROFS abled to find their solution. Hence, the invented theories are massively modified by prevailing theories (Das and Granados 2022; Garg and Rani 2019a; Ullah et al. 2020a).

5 Conclusion

In the presence of the Cq-ROFS, we demonstrated several important ideas and discussed their themes with the help of examples. The major contribution of this study is to analyze the conception of VCSMs and generalized VCSMs in the consideration of Cq-ROFSs and illustrate their properties. Additionally, a lot of special cases of the invented measures are demonstrated to expand the superiority of the investigated works. In the consideration of the generalized VCSMs using complex q-rung orthopair fuzzy information, a medical diagnosis is illustrated to determine the brain carcinoma in the human body. To illustrate the supremacy and dominance of the exposed works, various examples are illustrated. Finally, we determined the advantages and sensitive analysis of the initiated measures to illustrate the rationality and dominance of the developed measures.

There also exists a lot of ambiguity that occurred in the prevailing theories, we tried to give some practical examples, in which places the invented measures based on Cq-ROFS have been unsuccessful. For example, if an individual faced data in the form, which includes three sorts of data, called truth, abstinence, and falsity grade with a technique that the sum of the triplet should be lies among in unit interval. For this, in the consideration of the Arithmetic Optimization Algorithm (Abualigah et al. 2021a), Dwarf Mongoose Optimization Algorithm (Agushaka et al. 2022), Aquila optimizer: a novel meta-heuristic optimization algorithm (Abualigah et al. 2021b), Reptile Search Algorithm (RSA): a nature-inspired meta-heuristic optimizer (Abualigah et al. 2022), Ebola Optimization Search Algorithm: a new nature-inspired meta-heuristic algorithm (Oyelade et al. 2022), T-spherical (Khan et al. 2023; Ullah et al. 2020b), complex spherical (Ashraf et al. 2022), and complex T-spherical fuzzy sets (Ali et al. 2020; Liu et al. 2020b; Badi et al. 2022) and other uncertainty theories (Puška et al. 2023; Więckowski et al. 2023; Zhou et al. 2022; Karamaşa et al. 2021), we employed several measures in the region of the above ideas.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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