Alexandria Engineering Journal (2023) 71, 535–550

HOSTED BY

ELSEVIER

Alexandria University

Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com

ORIGINAL ARTICLE

Maclaurin symmetric mean aggregation operators based on novel Frank T-norm and T-conorm for intuitionistic fuzzy multiple attribute group decision-making

Amir Hussain [a], **Haolun Wang** [b,**], **Kifayat Ullah** [a], **Harish Garg** [c,d,e,f,*], **Dragan Pamucar** [g,h]

[a] *Department of Mathematics, Riphah International University Lahore, Lahore 54000, Pakistan*
[b] *School of Economics and Management, Nanchang Hongkong University, China*
[c] *School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala 147004, Punjab, India*
[d] *Department of Mathematics, Graphic Era Deemed to be University, Dehradun 248002, Uttarakhand, India*
[e] *Applied Science Research Center, Applied Science Private University, Amman 11931, Jordan*
[f] *College of Technical Engineering, The Islamic University, Najaf, Iraq*
[g] *Faculty of Organizational Sciences, University of Belgrade, 11000 Belgrade, Serbia*
[h] *College of Engineering, Yuan Ze University, Taiwan*

Received 22 December 2022; revised 2 March 2023; accepted 20 March 2023

KEYWORDS

Frank t-norm and t-conorm;
Intuitionistic fuzzy set;
Maclaurin symmetric mean;
Multi-attribute decision-making;
Aggregation operators

Abstract Multi-attribute group decision-making (MAGDM) is an interesting technique to find the most optimal alternative among comparative alternatives. Several authors put forward to MAGDM by introducing different fuzzy frameworks and also different tools to deal with fuzzy information. Intuitionistic fuzzy set (IFS) is the fuzzy framework that deals with the uncertainty in MAGDM. Due to their flexibility and generality, Frank t-norm (FTNM) and t-conorm (FTCNM) play an essential role in information fusion. Moreover, as the generalization of some mean operators, the Maclaurin symmetric mean (MSM) operator considers the relationship between multi-criteria arguments, especially in MAGDM. This article aims to develop some MSM aggregation operators (AOs) for the intuitionistic fuzzy set (IFS) based on FTNM and FTCNM and to apply newly developed AOs in the MAGDM. To utilize the MAGDM algorithm, first, we defined the MSM by using the FTNM and FTCNM in the environment of IFS. Then we proposed intuitionistic fuzzy (IF) Frank MSM (IFFMSM) and IF Frank weighted MSM (IFFWMSM) operators. Then, the fundamental properties of these AOs are stated and proved.

* Corresponding author at: School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala 147004, Punjab, India.
**Corresponding author at: School of Economics and Management, Nanchang Hongkong University, China.
E-mail addresses: hlwang71162@nchu.edu.cn (H. Wang), kifayat.khan.dr@gmail.com (K. Ullah), harish.garg@thapar.edu, harishg58iitr@gmail.com (H. Garg).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

https://doi.org/10.1016/j.aej.2023.03.063
1110-0168

Then, the strategy is given that accounts for the application of the newly developed family of AOs. Further, freshly defined operators are applied to the MAGDM problem with the help of an example where the risk factors of the construction industry are assessed. To cope with the significance, the proposed AOs are compared with some existing AOs. This study also addresses the variation of these AOs' behavior based on the interpretation of sensitive parameters.

1. Introduction

The MAGDM helps to choose an optimal alternative from the provided list considering some attributes. The MAGDM is more reliable than the multi-attribute decision-making (MADM) because, in MAGDM, the result is cumulatively based on experts' opinions. In contrast, in the MADM, the result is based on the opinion of only one expert. Hence the MAGDM is considered a more effective and efficient technique in decision-making. The MAGDM has been applied in the areas of energy [2,17,31], business [19,36], medical science [1,12,26], etc. Moreover, due to the complexity and ambiguity of the information, the result obtained by decision-making may not be expressed with proper mathematical modeling, especially in the case of human opinion. To cope with the fuzziness and complexity of information, [43] introduced an exciting idea of the fuzzy set (FS), where he described an object as a member of phenomena or set in the form of a membership grade (MG) by taking the values of this MG from $[0,1]$. After the introduction of FSs, a new direction was set for mathematicians by [9] to express the ambiguity in uncertain information. [9] thought to generalize the idea of FS by adding grades to show the non-membership of an object to a phenomenon or simply a set called the non-membership grade (NG). Consequently, [9] proposed the idea of intuitionistic FS (IFS) with the NG. The framework of the IFS covered the complexity and the vagueness of the information more accurately than the FS. Consequently, scholars applied the IFS to cope with the problems due to the complexity and ambiguity of the information. For example, AOs based on IFS [3], complex intuitionistic fuzzy (CIF) classes [5], IFS in graph theory [8], IFS in group theory [4], IFS in distance measure [10], IFS in pattern recognition [11], etc., are some developments in various fields depending upon the IFS.

In the MAGDM, the role of AOs is very significant because the AOs aggregate the information comprehensively [16]. Depending on the AOs, many advantageous methods to aggregate the information have been developed. For example, the authors [27,20,46] introduced some important AOs. Unlike traditional AOs, these methods comprehensively aggregate the information of alternatives. In MADM and MAGDM, the AOs have been becoming the most attractive topic to research for a few years due to their vast applicability. For example, AOs developed by the authors [13], AOs by [15], and [14] in the different fuzzy frameworks are some AOs developed in the past few years. Some techniques to deal with the MAGDM can be found in [22–25].

The AOs mentioned above aggregate multiple numbers into a single number. MSM operators defined by Recently, a few researchers have developed various types of AOs that aggregate multiple numbers depending on the different attributes by considering their relationship between them. For example, AOs developed by [41,42,35,39] are some AOs to aggregate fuzzy information considering the relationship between attributes. For interval-valued IFS [25] introduced power MSM AOs which aggregates information in the form of the intervals. The limitation of these AOs is that they are based on the traditional operational laws. In addition, some researchers have applied various operational laws to acquire better and flexible results than the aforementioned AOs. For example, t-norm (TNM) and t-conorm (TCNM) are used in power AOs by [6,33] used Dombi TNM, and TCNM, [18] used the Hamacher TNM and TCNM, [38,47] used Einstein TNM and TCNM, and [44,45] used FTNM, and FTCNM and so on.

IFS has become a popular framework among researchers to deal with the ambiguity and uncertainty in information. The main points of the motivations for preparing this article are given as;

1. Frank's family of operational laws [30] are very flexible and reliable due to the involvement of parameters and are widely applied in MAGDM by researchers as in [44,45] due to their flexibility.
2. IFS is the framework to handle the fuzziness and ambiguity in information by describing an object by two grades, MG and NMG, as mentioned above; consequently, IFS has provided better results than the crisp set and FS while used in MAGDM in various fields.
3. MSM is the operator that contributes to decision-making by aggregating the information of alternatives and considering their relationship. MSM has also been an exciting tool to aggregate the data and provide better results because it deals with the attributes by considering their inter-relationship.
4. The existing MSM operators for IFS based on the traditional operational laws. In this study we develop MSM operators based on the flexible operational laws i.e., FTNM and FTCNM.

There are five sections in this article. Section 2 states the fundamental concepts for a better understanding of the article. Then in Section 3, the newly proposed IFFMSM AOs are developed, and their properties are stated and proved. In Section 4, the IFFWMSM operator is designed, and its basic properties are stated. Section 5 consists of the application of the proposed AOs with the help of an example. In Section 5, for the significance of our proposed work, we have also compared it with existing AOs. Finally, Section 6 concludes our detailed discussion.

2. Preliminaries

In this section, we elaborate on some fundamental concepts of IFS, MSM, FTN, and FTCNM to better understand this article.

Definition 1. [9] *Let $\ddot{Y}$ be the universal set. Then an IFS $\dot{A}$ is defined as,*

$$\dot{A} = \{\langle q, \varkappa_{\dot{A}}(q), \hbar_{\dot{A}}(q)\rangle | q \in \ddot{Y}\}$$

Where $\varkappa_{\dot{A}} : \ddot{Y} \to [0,1], \hbar_{\dot{A}} : \ddot{Y} \to [0,1]$ are the MG and NG, respectively, with $\varkappa_{\dot{A}} + \hbar_{\dot{A}} \in [0,1]$. The hesitancy degree is determined by $1 - \varkappa_{\dot{A}} - \hbar_{\dot{A}}$.

Definition 2. [9] *Assume $\mathscr{H} = (\varkappa_{\dot{A}}, \hbar_{\dot{A}})$ be an IFV. The score function of $\mathscr{H}$ is defined as follows,*

$$\mathrm{Sc}(\mathscr{H}) = \varkappa_{\dot{A}} - \hbar_{\dot{A}} \tag{1}$$

Where $\mathrm{Sc}(\mathscr{H}) \in [-1,1]$. Smaller the $\mathrm{Sc}(\mathscr{H})$ is the smaller the IFV $\mathscr{H}$ is.

It was found that some IFVs could not be ranked with the help of the score function defined above. [17] gave the idea of the grade of accuracy (GA), which is described below.

Definition 3. *Assume $\mathscr{H} = (\varkappa_{\dot{A}}, \hbar_{\dot{A}})$ be an IFV. The GA of $\mathscr{H}$ is defined as the following,*

$$GA(\mathscr{H}) = \varkappa_{\dot{A}} + \hbar_{\dot{A}} \tag{2}$$

$$\mathrm{GA}(\mathscr{H}) \in [0,1]$$

Definition 4. [30] *The Frank TNM and TCNM are mappings such that $[0,1]^2 \to [0,1]$ given below;*

$$T_F(r,s) = Log_{\aleph}\left(1 + \frac{(\aleph^r - 1)(\aleph^s - 1)}{\aleph - 1}\right), r, s \in [0,1], \aleph > 1$$

$$S_F(r,s) = 1 - Log_{\aleph}\left(1 + \frac{(\aleph^{1-r} - 1)(\aleph^{1-s} - 1)}{\aleph - 1}\right), r, s \in [0,1], \aleph > 1$$

Definition 5. *Assume $\mathscr{H}_1 = (\varkappa_{\dot{A}1}, \hbar_{\dot{A}1})$, $\mathscr{H}_2 = (\varkappa_{\dot{A}2}, \hbar_{\dot{A}2})$ and $\mathscr{H} = (\varkappa_{\dot{A}}, \hbar_{A})$ be three IFVs and $n > 0$, the operations of IFVs based on Frank TNM and TCNM are defined as follows,*

$$\mathscr{H}_1 \oplus_F \mathscr{H}_2 = \begin{pmatrix} 1 - Log_{\aleph}\left(1 + \frac{(\aleph^{1-\varkappa_{\dot{A}1}} - 1)(\aleph^{1-\varkappa_{\dot{A}2}} - 1)}{\aleph - 1}\right), \\ Log_{\aleph}\left(1 + \frac{(\aleph^{\hbar_{\dot{A}1}} - 1)(\aleph^{\hbar_{\dot{A}2}} - 1)}{\aleph - 1}\right) \end{pmatrix}$$

$$\mathscr{H}_1 \otimes_F \mathscr{H}_2 = \begin{pmatrix} Log_{\aleph}\left(1 + \frac{(\aleph^{\varkappa_{\dot{A}1}} - 1)(\aleph^{\varkappa_{\dot{A}2}} - 1)}{\aleph - 1}\right), \\ 1 - Log_{\aleph}\left(1 + \frac{(\aleph^{1-\hbar_{\dot{A}1}} - 1)(\aleph^{1-\hbar_{\dot{A}1}} - 1)}{\aleph - 1}\right) \end{pmatrix}$$

$$n.\mathscr{H} = \left(1 - Log_{\aleph}\left(1 + \frac{(\aleph^{1-\varkappa} - 1)^n}{(\aleph - 1)^{n-1}}\right), Log_{\aleph}\left(1 + \frac{(\aleph^{\hbar} - 1)^n}{(\aleph - 1)^{n-1}}\right)\right)$$

$$\mathscr{H}^n = \left(Log_{\aleph}\left(1 + \frac{(\aleph^{\varkappa} - 1)^n}{(\aleph - 1)^{n-1}}\right), 1 - Log_{\aleph}\left(1 + \frac{(\aleph^{1-\hbar} - 1)^n}{(\aleph - 1)^{n-1}}\right)\right) \tag{3}$$

Definition 6. [28] *Assume $h_m (m = 1,2,3,\cdots,n)$ be the collection of positive real numbers. Then the MSM operator can be given as follows,*

$$MSM(h_1, h_2, h_3, \cdots h_n) = \left(\frac{\sum_{1 \le m_1 < m_2 < m_3 < \cdots < m_w \le n} \prod_{l=1}^{w} h_{m_l}}{C_w^n}\right)^{1/w} \tag{4}$$

Where, C_w^n is the binomial coefficient, $1 \le m_1 < m_2 < m_3 < \cdots < m_w \le n$ represents *thew − tuple* combination of the positive real numbers.

3. Frank Maclaurin symmetric mean operator for IFVs

This section consists of the development of the IFFMSM operator and, which is based on the Frank operations on IFVs. In this section, we also study some desirable properties of IFMSM operator.

Definition 7. *Let $\mathscr{H}_i (i = 1,2,3\cdots,n)$ be the collection of the IFVs,$w = 1,2,3,\cdots,n$. Then IFFMSM: $\ddot{Y}^n \to \ddot{Y}$ an operator is defined as,*

$$\mathrm{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \left(\frac{\bigoplus_{F\, 1 \le i_1 < i_2 < i_3 < \cdots < i_w \le n} \bigotimes_{F\, j=1}^{w} \mathscr{H}_{i_j}}{C_w^n}\right)^{1/w} \tag{5}$$

Where, C_w^n is the binomial coefficient, $1 \le i_1 < i_2 < i_3 < \cdots < i_w \le n$ represents *thew − tuple* combination of the IFVs.

Based on the Frank operations defined above, we can prove some fundamental properties of the IFFMSM operator. In Theorem 1, we demonstrate the aggregated value obtained by IFFMSM is again an IFV.

Theorem 1. *Let $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1,2,3,\cdots,n)$ be the collection of IFVs and $w = 1,2,3,\cdots,n$. Then an IFV is obtained as aggregated value by the IFFMSM operator and.*

Which is an IFV. Hence the proof is completed.

$$\mathrm{IFFMSM}(\mathscr{H}_1,\mathscr{H}_2,\mathscr{H}_3,\cdots\mathscr{H}_n)==\left(\begin{array}{c}Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-\left(Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right),\\ 1-Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)\end{array}\right)$$

Proof: To prove this theorem, we calculate $\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}$ first, as follows,

$$\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}=\left(Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{\varkappa_i}-1\right)\right)(\aleph-1)^{1-n}\right),1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i}-1\right)\right)(\aleph-1)^{1-n}\right)\right)$$

Which is an IFV. Now, we calculate $\oplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n}\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}$ as follows,

$$\oplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n}\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}=\left(\begin{array}{c}1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-\left(Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right),\\ Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)\end{array}\right)$$

$\oplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n}\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}$ is also clearly an IFV. Now, we calculate $\left(\frac{\oplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n}\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}}{C_w^n}\right)^{1/w}$ as follows,

$$\left(\frac{\oplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n}\otimes_{F\,j=1}^{\;w}\mathscr{H}_{i_j}}{C_w^n}\right)^{1/w}=\left(\begin{array}{c}Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-\left(Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right),\\ 1-Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right)\end{array}\right)$$

Theorem 2. *(Idempotency): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1, 2, 3, \cdots, n)$ *be the collection of IFVs, and if* $\mathscr{H}_i = (\varkappa_i, \hbar_i) = \mathscr{H} = (\varkappa, \hbar)\forall(i = 1, 2, 3, \cdots, n)$. *Then* $\text{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \mathscr{H}$ *is called the idempotent.*

Proof: As $\mathscr{H}_i = (\varkappa_i, \hbar_i) = (\varkappa, \hbar) = \mathscr{H}\forall(i = 1, 2, 3, \cdots, n)$, by the Theorem 1, we have.

$$\text{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \left(\begin{array}{c} Log_{\aleph}\left(1 + \left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-\left(Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right), \\ 1 - Log_{\aleph}\left(1 + \left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right) \end{array} \right)$$

$$= \left(\begin{array}{c} Log_{\aleph}\left(1 + \left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-\left(Log_{\aleph}\left(1+\frac{\left(\aleph^{\varkappa}-1\right)^{n}}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right), \\ 1 - Log_{\aleph}\left(1 + \left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\left(\aleph^{1-\hbar}-1\right)^{n}}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right) \end{array} \right)$$

$$= \left(\begin{array}{c} Log_{\aleph}\left(1 + \left(\aleph^{\varkappa} - 1\right)^{\frac{1}{w}}(\aleph - 1)^{1-\frac{1}{w}}\right), \\ 1 - Log_{\aleph}\left(1 + \left(\aleph^{1-\hbar} - 1\right)^{\frac{1}{w}}(\aleph - 1)^{1-\frac{1}{w}}\right) \end{array} \right)$$

$$= (\varkappa, \hbar) = \mathscr{H}$$

Hence the proof is completed.

Theorem 3. *(Monotonicity): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)$*and* $\mathscr{H}_i^a = \left(\varkappa_i^a, \hbar_i^a\right)(i = 1, 2, 3, \cdots, n)$ *be the collections of IFVs and* *if* $\mathscr{H}_i \geq \mathscr{H}_i^a$ *for all* $(i = 1, 2, 3, \cdots, n)$. *Then* $\mathrm{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \geq \mathrm{IFFMSM}(\mathscr{H}_1^a, \mathscr{H}_2^a, \mathscr{H}_3^a, \cdots \mathscr{H}_n^a)$.

Proof: As stated that $\mathscr{H}_i \geq \mathscr{H}_i^a$ so, for IFVs $\varkappa_i \geq \varkappa_i^a$ and $\hbar_i \leq \hbar_i^a$. We can write.

$$Log_{\aleph}\left(1 + \left(\prod_{i=1}^{n}\left(\aleph^{\varkappa_i} - 1\right)\right)(\aleph - 1)^{1-n}\right)$$

$$\geq Log_{\aleph}\left(1 + \left(\prod_{i=1}^{n}\left(\aleph^{\varkappa_i^a} - 1\right)\right)(\aleph - 1)^{1-n}\right)$$

And.

$$1 - Log_{\aleph}\left(1 + \left(\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i} - 1\right)\right)(\aleph - 1)^{1-n}\right)$$

$$\leq 1 - Log_{\aleph}\left(1 + \left(\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i^a} - 1\right)\right)(\aleph - 1)^{1-n}\right)$$

Further, we have.

$$1 - Log_{\aleph}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\aleph^{1-\left(Log_{\aleph}\left(1 + \frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa_i} - 1\right)}{(\aleph - 1)^{w-1}}\right)\right)} - 1\right)(\aleph - 1)^{1-w}\right) \geq 1$$

$$- Log_{\aleph}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\aleph^{1-\left(Log_{\aleph}\left(1 + \frac{\prod_{i=1}^{n}\left(\aleph^{\varkappa_i^a} - 1\right)}{(\aleph - 1)^{w-1}}\right)\right)} - 1\right)(\aleph - 1)^{1-w}\right)$$

And.

$$Log_{\aleph}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\aleph^{1-\left(Log_{\aleph}\left(1 + \frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i} - 1\right)}{(\aleph - 1)^{w-1}}\right)\right)} - 1\right)(\aleph - 1)^{1-w}\right)$$

$$\leq Log_{\aleph}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\aleph^{1-\left(Log_{\aleph}\left(1 + \frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i^a} - 1\right)}{(\aleph - 1)^{w-1}}\right)\right)} - 1\right)(\aleph - 1)^{1-w}\right)$$

Then we have.

$$\boldsymbol{Log}\left(1 + \left(\left(1 - \boldsymbol{Log}\left(1 + \left(\left(1 - \boldsymbol{Log}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\left(1 - \left(\boldsymbol{Log}\left(1 + \frac{\prod_{i=1}^{n}(\varkappa_i - 1)}{(-1)^{w-1}}\right)\right)\right) - 1\right)(-1)^{1-w}\right)\right) - 1\right)^{\frac{1}{C_w^n}}(-1)^{1-\frac{1}{C_w^n}}\right)\right) - 1\right)^{\frac{1}{w}}(-1)^{1-\frac{1}{w}}\right)$$

$$\geq \boldsymbol{Log}\left(1 + \left(\left(1 - \boldsymbol{Log}\left(1 + \left(\left(1 - \boldsymbol{Log}\left(1 + \prod_{1 \leq i_1 < i_2 < i_3 < \cdots < i_w \leq n}\left(\left(1 - \left(\boldsymbol{Log}\left(1 + \frac{\prod_{i=1}^{n}\left(\varkappa_i^a - 1\right)}{(-1)^{w-1}}\right)\right)\right) - 1\right)(-1)^{1-w}\right)\right) - 1\right)^{\frac{1}{C_w^n}}(-1)^{1-\frac{1}{C_w^n}}\right)\right) - 1\right)^{\frac{1}{w}}(-1)^{1-\frac{1}{w}}\right)$$

And.

$$1-Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right)$$

$$\le 1-Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\left(\aleph^{Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{\left(1-Log_{\aleph}\left(1+\frac{\prod_{i=1}^{n}\left(\aleph^{1-\hbar_i^{a}}-1\right)}{(\aleph-1)^{w-1}}\right)\right)}-1\right)(\aleph-1)^{1-w}\right)}-1\right)^{\frac{1}{C_w^n}}(\aleph-1)^{1-\frac{1}{C_w^n}}\right)}-1\right)^{\frac{1}{w}}(\aleph-1)^{1-\frac{1}{w}}\right)$$

This completes the proof.

Theorem 4. *(Boundedness): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1, 2, 3, \cdots, n)$ *be the collections of IFVs, and if* $\mathscr{H}^- = (\min\varkappa_i, \max\hbar_i), \mathscr{H}^+ = (\max\varkappa_i, \min\hbar_i)$*. Then.*

$\mathscr{H}_i^- \le \mathrm{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \le \mathscr{H}_i^+$

Proof: Since $\varkappa^- = \min(\varkappa_i)$ and $\hbar^+ = \max(\hbar_i)$. Therefore, we can write $\varkappa^- \le \varkappa_i \le \varkappa^+, \hbar^- \le \hbar_i \le \hbar^+$ for all $i = 1, 2, 3, \cdots, n$. Therefore, by the Theorem 3 and 4, we can write.

$\mathscr{H}_i^- \le \mathrm{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \le \mathscr{H}_i^+$

Hence the proof is completed.

Example 1. *Consider* $\mathscr{H}_1 = (0.1, 0.2), \mathscr{H}_2 = (0.4, 0.5)$, *and* $\mathscr{H}_3 = (0.3, 0.5)$ *are three IFVs. Here,* $\varkappa^- = 0.1$ *and* $\hbar^+ = 0.5$*. Hence,* $\mathscr{H}^- = (0.1, 0.5)$ *and similarly* $\mathscr{H}^+ = (0.4, 0.2)$*. Now, we aggregate these values by proposed operators as follows.*

$\mathrm{IFFMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3) = (0.4, 0.3)$

Hence, cleared from Example 1 the IFFMSM operator is monotonic. It is important to note that the monotonicity can not be proved for the score values of provided IFVs.

4. Intuitionistic fuzzy frank weighted maclaurin symmetric mean operator

The operator proposed in Section 3 aggregates the attributes collectively. However, it cannot consider the self-importance of various aggregated attributes. Therefore, we IFFWMSM operator to overcome this problem. Note that we will use B for the provided weight vector where $B = \{\delta_i\}, i = 1, 2, \cdots n$ and $\sum_{i=1}^{n}\delta_i = 1$ in the further discussion unless otherwise stated.

Definition 8. *Let* $\mathscr{H}_i(i = 1, 2, 3 \cdots, n)$ *be the collection of the IFVs,* $w = 1, 2, 3, \cdots, n$ *and* B *be the weight vector. Then IFFMSM:* $\ddot{Y}^n \to \ddot{Y}$ *the operator is defined as,*

$$\mathrm{I}FFMSM(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \left(\frac{\bigoplus_{F\,1\le i_1<i_2<i_3<\cdots<i_w\le n} \bigotimes_{F\,j=1}^{w} \delta_{i_j}\mathscr{H}_{i_j}}{C_w^n}\right)^{1/w} \quad (6)$$

Where, C_w^n is the binomial coefficient, $1 \le i_1 < i_2 < i_3 < \cdots < i_w \le n$ represent w – tuple combination of the IFVs.

Based on the Frank operations defined in Eqn. (3), we can prove some fundamental properties of the IFFWMSM operator. In Theorem 1, we demonstrate the aggregated value obtained by IFFWMSM operator is an IFV.

Theorem 5. *Let $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1, 2, 3, \cdots, n)$ be the collection of IFVs and $w = 1, 2, 3, \cdots, n$. Then the value obtained as aggregated value by the IFFWMSM operator is also an IFVIFV which is given as follows,*

$$\mathrm{IFFWMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \left(\begin{array}{c} \mathrm{Log}_{\aleph}\left(1 + \dfrac{\left(\aleph^{1-\mathrm{Log}_{\aleph}\left(1+\frac{\left(\aleph^{1-\mathscr{E}}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right)}-1\right)^{\frac{1}{w}}}{(\aleph-1)^{w-1}}\right), \\ 1-\mathrm{Log}_{\aleph}\left(1 + \dfrac{\left(\aleph^{1-\mathrm{Log}_{\aleph}\left(1+\frac{\left(\aleph^{\varphi}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right)}-1\right)^{\frac{1}{w}}}{(\aleph-1)^{\frac{1}{w}-1}}\right) \end{array} \right)$$

Where to avoid the length of the equations we assumed.

$$\mathscr{E} = 1 - Log_{\aleph}\left(1 + \prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\varkappa_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right)$$

And.

$$\varphi = Log_{\aleph}\left(1 + \prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right)$$

Proof: To prove this theorem, we calculate $\bigotimes_{F\, j=1}^{w} \delta_{i_j}\mathscr{H}_{i_j}$ first, as follows,

$$\bigotimes_{F\, j=1}^{w} \delta_{i_j}\mathscr{H}_{i_j} = \left(\begin{array}{c} Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\varkappa_i}-1\right)^{\delta_{i_j}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right), \\ 1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\delta_{i_j}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right) \end{array} \right)$$

Which is an IFV. Now, we calculate $\underset{1\le i_1<i_2<i_3<\cdots<i_w\le n}{\oplus_F}\overset{w}{\underset{j=1}{\otimes_F}}\delta_{i_j}\mathscr{H}_{i_j}$ as follows,

$$\underset{1\le i_1<i_2<i_3<\cdots<i_w\le n}{\oplus_F}\overset{w}{\underset{j=1}{\otimes_F}}\delta_{i_j}\mathscr{H}_{i_j}=\left(\begin{array}{c}1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\kappa_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right),\\ Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right)\end{array}\right)$$

To avoid the extra length of the equations we let.

$$\mathscr{E}=1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\kappa_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right)$$

And.

$$\varphi=Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right)$$

Further we have.

$$\frac{\underset{1\le i_1<i_2<i_3<\cdots<i_w\le n}{\oplus_F}\overset{w}{\underset{j=1}{\otimes_F}}\delta_{i_j}\mathscr{H}_{i_j}}{C_w^n}=\left(\begin{array}{c}1-Log_{\aleph}\left(1+\frac{\left(\aleph^{1-\mathscr{E}}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right),\\ Log_{\aleph}\left(1+\frac{\left(\aleph^{\varphi}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right)\end{array}\right)$$

Finally,

$$\left(\frac{\underset{1\le i_1<i_2<i_3<\cdots<i_w\le n}{\oplus_F}\overset{w}{\underset{j=1}{\otimes_F}}\delta_{i_j}\mathscr{H}_{i_j}}{C_w^n}\right)^{\frac{1}{w}}=\left(\begin{array}{c}Log_{\aleph}\left(1+\frac{\left(\aleph^{1-Log_{\aleph}\left(1+\frac{\left(\aleph^{1-\mathscr{E}}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right)}-1\right)^{\frac{1}{w}}}{(\aleph-1)^{w-1}}\right),\\ 1-Log_{\aleph}\left(1+\frac{\left(\aleph^{1-Log_{\aleph}\left(1+\frac{\left(\aleph^{\varphi}-1\right)^{C_w^n}}{(\aleph-1)^{C_w^n-1}}\right)}-1\right)^{\frac{1}{w}}}{(\aleph-1)^{\frac{1}{w}-1}}\right)\end{array}\right)$$

Which, is an IFV. Hence the proof is completed.

Theorem 6. *(Idempotency): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1, 2, 3, \cdots, n)$ *be the collection of IFVs, and if* $\mathscr{H}_i = (\varkappa_i, \hbar_i) = (\varkappa, \hbar) = \mathscr{H} \forall (i = 1, 2, 3, \cdots, n)$. *Then* $\mathrm{IFFWMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) = \mathscr{H}$ *is called the idempotency*.

Proof: As $\mathscr{H}_i = (\varkappa_i, \hbar_i) = \mathscr{H} \forall (i = 1, 2, 3, \cdots, n)$. Then we have.

$$\mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j} = \begin{pmatrix} Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\kappa_i}-1\right)^{\delta_{i_j}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right), \\ 1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\delta_{i_j}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right) \end{pmatrix} = (\varkappa, \hbar)$$

Further,

$$\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j} = \begin{pmatrix} 1-Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{1-\kappa_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right), \\ Log_{\aleph}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\prod_{i=1}^{n}\left(\aleph^{1-Log_{\aleph}\left(1+\left(\aleph^{\hbar_i}-1\right)^{\frac{1-\delta_{i_j}}{\delta_{i_j}}}(\aleph-1)^{1-\delta_{i_j}}\right)}-1\right)\right)(\aleph-1)^{-1}\right)}-1\right)(\aleph-1)^{1-w}\right) \end{pmatrix}$$

$= (\varkappa, \hbar)$

Finally,

$$\left(\frac{\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}}{C_w^n}\right)^{\frac{1}{w}} = (\varkappa, \hbar)$$

Hence, proof is completed.

Theorem 7. *(Monotonicity): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)$ *and* $\mathscr{H}_i^a = \left(\varkappa_i^a, \hbar_i^a\right)(i = 1, 2, 3, \cdots, n)$ *be the collections of IFVs, and if* $\mathscr{H}_i \ge \mathscr{H}_i^a$ *for all* $(i = 1, 2, 3, \cdots, n)$. *Then.*

$$\mathrm{IFFWMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \ge \mathrm{IFFWMSM}\left(\mathscr{H}_1^a, \mathscr{H}_2^a, \mathscr{H}_3^a, \cdots \mathscr{H}_n^a\right)$$

Proof: As stated that $\mathscr{H}_i \ge \mathscr{H}_i^a$ so, for IFVs $\varkappa_i \ge \varkappa_i^a$ and $\hbar_i \le \hbar_i^a$. We have.

$$\mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j} \ge \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}^a$$

Further,

$$\frac{\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}}{C_w^n} \ge \frac{\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}^a}{C_w^n}$$

Finally,

$$\left(\frac{\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}}{C_w^n}\right)^{\frac{1}{w}} \ge \left(\frac{\mathop{\oplus_F}_{1\le i_1<i_2<i_3<\cdots<i_w\le n} \mathop{\otimes_F}_{j=1}^{w} \delta_{i_j} \mathscr{H}_{i_j}^a}{C_w^n}\right)^{\frac{1}{w}}$$

Hence, proof is completed.

Theorem 8. *(Boundedness): Let* $\mathscr{H}_i = (\varkappa_i, \hbar_i)(i = 1, 2, 3, \cdots, n)$ *be the collections of IFVs, if* $\mathscr{H}^- = \min(\mathscr{H}_i), \mathscr{H}^+ = \max(\mathscr{H}_i)$. *Then.*

$$\mathscr{H}_i^- \le \mathrm{IFFWMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \le \mathscr{H}_i^+$$

Proof: Since and $\hbar^+ = \max(\hbar_i)$. Therefore, we can write $\varkappa^- \le \varkappa_i \le \varkappa^+, \hbar^- \le \hbar_i \le \hbar^+$ for all $i = 1, 2, 3, \cdots, n$. Therefore, by the Theorem 6 and 7, we can write.

$$\mathscr{H}_i^- \le \mathrm{IFFWMSM}(\mathscr{H}_1, \mathscr{H}_2, \mathscr{H}_3, \cdots \mathscr{H}_n) \le \mathscr{H}_i^+$$

Hence the proof is completed.

5. Application of proposed approach in MAGDM

This section involves the methodology to apply the proposed IFFWMSM operator to the decision-making problems having the fuzzy information in the form of IFVs. An example has also been provided in this section to discuss the application in the field of risk management for the construction industry. In the last of this section, the proposed IFFWMSM operator is compared with the existing AOs for significance of the proposed AOs.

Consider the list of n alternatives $Z = \{\zeta_1, \zeta_2, \cdots, \zeta_n\}$ from which anyone alternative has to be selected based on m attributes $B = \{\beta_1, \beta_2, \cdots, \beta_m\}$ having the weights from $\Psi = \{\psi_1, \psi_2, \cdots, \psi_m\}$ with $\sum_{i=1}^{m} \psi_i \forall i = 1, 2, 3, \cdots, m.$ Let $K = \{\kappa_1, \kappa_2, \cdots, \kappa_j\}$ be the weights of j experts and $T^s = [r^s{}_{ij}]_{m \times n}$ be the decision matrix where $r^s{}_{ij} = \left(\varkappa^s_{ij}, \hbar^s_{ij}\right)$ be the IF information provided by the decision expert s in the form of IFVs.

Based on the developed IFFWMSM operator the, the decision-making process is detailed below.

Step 1 Normalize the given IF information.

There are two types of attributes in general, i.e., benefit and cost. The impact of different attributes must be same before their aggregation. To do so, we change the cost type attribute in the benefit type. The normalization of the data is given below.

$$(T^s)^c = \begin{cases} \left(\varkappa^s_{ij}, \hbar^s_{ij}\right) forbenefitattribute \\ \left(\hbar^s_{ij}, \varkappa^s_{ij}\right) forcostattribute \end{cases}$$

Step 2 Aggregate the IF information with the help of the proposed IFFWMSM operator (given in Eq. (6)) to obtain the individual preferences values from the decision matrices provided.
Step 3 Apply the IFFMSM operator (given in Eq. (5)) to obtain the collective preference values.
Step 4 Calculate the score values with the help of Eq. (1) of the obtained collective preferences values.
Step 5 Rank the alternatives.

Now, we apply our proposed study to the real-life problem. The construction industry faces many risks that are affected by its growth. Some of the major examples are political instability, poor economic policies, poor trade policy, unforeseen immediate, and so on. The major problem is that there is no measurement tool to measure the effects of these factors; hence we cannot assess the major factor so that we can tawe steps to reduce the effects of that factor. Furthermore, every factor can be based on some attributes. Hence, we apply the proposed IFFWMSM operator to evaluate which factor is needed to be addressed on priority by considering and solving a MAGDM problem as in Example 2 in the following.

Example 2. *The construction industry is the back boon of the economy of any country. The well-developed countries have good construction industry because they focus on it and make policies for its improvement. However, the construction industry depends upon some attributes, and there also are some risk factors for it. In this example, we consider the problem of the risk management where some of the risk factors have to be examined and one of the most noticeable factors of these factors has to be determined.*

Assume that $1 \leq \partial_i \leq 4$ are considered risk factors, i.e., poor economic policies, political instability, poor trade policy, and unforeseen immediate for any construction industry after the initial screening based on some attributes $1 \leq \boldsymbol{\vartheta_i} \leq 4$ having weights $(0.2, 0.3, 0.25, 0.25)$. Consider there are three experts that are evaluating the risk factors based on abovementioned attributes. The weights of the attributes are $(0.24, 0.59, 0.17)$. The experts assign the values to each alternative in the form of the IFV corresponding to each attribute. The values assigned to each alternative from the decision experts in the form of the IFVs has been tabulated in Table 1, Table 2, and Table 3 as follows,

Step 1: In the above information, all the attributes are of the same type. Hence, we do not need the normalization because none of the attributes is the cost attribute.

Table 1 Decision matrix D1 provided by expert 1.

	∂_1		∂_2		∂_3		∂_4	
	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$
ϑ_1	0.4	0.4	0.1	0.2	0.3	0.5	0.4	0.3
ϑ_2	0.3	0.2	0.4	0.5	0.5	0.3	0.4	0.5
ϑ_3	0.5	0.2	0.2	0.5	0.2	0.5	0.3	0.4
ϑ_4	0.2	0.4	0.4	0.4	0.4	0.2	0.5	0.3

Table 2 Decision matrix D2 provided by expert 2.

	∂_1		∂_2		∂_3		∂_4	
	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$
ϑ_1	0.4	0.3	0.4	0.5	0.4	0.5	0.4	0.5
ϑ_2	0.3	0.2	0.3	0.4	0.3	0.4	0.3	0.4
ϑ_3	0.5	0.3	0.5	0.4	0.5	0.4	0.5	0.4
ϑ_4	0.4	0.4	0.3	0.4	0.5	0.3	0.4	0.5

Table 3 Decision matrix D3 provided by expert 3.

	∂_1		∂_2		∂_3		∂_4	
	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$
ϑ_1	0.4	0.5	0.3	0.5	0.2	0.4	0.3	0.5
ϑ_2	0.5	0.3	0.4	0.5	0.4	0.3	0.5	0.4
ϑ_3	0.4	0.5	0.5	0.3	0.5	0.4	0.4	0.5
ϑ_4	0.5	0.3	0.5	0.3	0.4	0.5	0.3	0.4

Step 2: We apply the IFFWMSM operator to aggregate all the individual values of attributes given in Tables 1- 3. The obtained collective preference values, which are given in Table 4 as follows,
Step 3: Then, we aggregate the obtained collective preference values obtained in Table 4 by using IFFMSM operator obtain the individual preference values and the results are given in Table 5,
Step 4: We evaluate the ranking order of the alternatives with the help of the score function. The ranking orders of the risk factors are given in Table 6, as follows,

As we obtain $sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$, hence $\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$, which means that ∂_1 is the most appropriate alternative. This means that the factor (poor economic policies) is a major risk to the construction industry.

5.1. Sensitivity analysis

The important characteristic of the FTNM and FTCNM is that these operations are very flexible due to the involvement of the parameter $\aleph$. But the results may vary with the value of this parameter when we change its value. In the following, we have observed the variations of the results in Table 7.

Table 6 Score values of risk factors.

$\boldsymbol{sc}(\partial_1)$	$\boldsymbol{sc}(\partial_2)$	$\boldsymbol{sc}(\partial_3)$	$\boldsymbol{sc}(\partial_4)$
0.648	0.375	0.540	0.431

We have examined the results obtained by our proposed IFFWMSM and IFFMSM operators by variating the values of $\aleph$. We vary the value of $\aleph$ from 2 to 50. The interesting results are tabulated in Table 7. At all the values of $\aleph$ we found the poor economic policies as the major risk factor for the construction industry. Hence, the IFFWMSM operator is stable after at all the values of $\aleph \geq 2$ and the decision-maker has a choice to select the value of $\aleph$ of their own choices which is the most important characteristic of the FTNM and FTCNM. We also can recommend the $\aleph \geq 2$ as the best value of the parameter to obtain stable results.

The pictorial representation of the analysis tabulated in Table 7 is below in Fig. 1.

Fig. 1 shows the variation of the score values of the risk factors (alternatives) on the different values of the parameter $\aleph$. On the vertical and horizontal axes, the values of $\aleph$ and

Table 4 Collective Preference Values obtained by IFFWMSM.

	∂_1		∂_2		∂_3		∂_4	
	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$
ϑ_1	0.54	0.28	0.48	0.02	0.59	0.04	0.58	0.04
ϑ_2	0.43	0.19	0.58	0.04	0.68	0.08	0.61	0.05
ϑ_3	0.63	0.36	0.60	0.05	0.66	0.07	0.61	0.05
ϑ_4	0.36	0.14	0.60	0.05	0.70	0.09	0.60	0.05

Table 5 Individual preferences values obtained from Table 4.

	∂_1		∂_2		∂_3		∂_4	
	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$	$\varkappa$	$\hbar$
$\mathbf{r_{ij}}$	0.752	0.104	0.816	0.440	0.882	0.342	0.842	0.411

Table 7 Variation in ranking orders of risk factors in IFFWMSM operator with parameter.

	Ranking of score values	Preference Orders
2	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
3	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
4	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
5	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
6	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
7	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
8	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
9	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
10	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
15	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
20	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
30	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
40	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
50	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$

obtained score values of risk factors are plotted respectively. It is clear from Fig. 1 that the score values of these risk factors change linearly on the different values of parameter $\aleph$. We can also observe from the graph that the ranking patterns of the risk factors remain the same throughout the graph. Hence the variation of the parameter $\aleph$ does not affect the ranking (choices) of the risk factors.

5.2. Comparison with pre-existing operators

[40] developed the geometric aggregation operator (IFG) for IFS to solve the MAGDM problem. Zhang et al. [44] studied the application of Frank operations in the framework of the IFS and then solved the MAGDM problem. Qin & Liu [32] developed some AOs for IFS based on MSM to solve the MAGDM problem. Senapati et al. [34] introduced AOs based on Aczel-Alsina TNM and TCNM (IFAAWA) for IFS. However, the proposed AOs in this study are the most reliable because they based on the most generalized form of the mean operator i.e., MSM operator and hence they aggregate attributes by keeping their relationship preservative. Moreover, the developed model is obtained based on the Frank's operational laws which are more flexible and generalized. Consequently, the developed model is more advantageous method to aggregate information.

The proposed approach has been compared with the pre-existing approaches mentioned above for the justification of the proposed approach. The results obtained by the proposed approach and the pre-existing approaches have been tabulated in the following in Table 8. The comparative results have been graphically represented as well in Fig. 1. Some of the key points of this comparison have been listed below.

With the help of the proposed approach IFFWMSM operator, the best alternative obtained is ∂_1. Similarly, the most appropriate alternative obtained by the IFWG operator is also ∂_1. The most appropriate alternative obtained by both operators in Zhang et al. [44], Xu & Yager [40] and Qin & Liu [32] is the also ∂_1 which shows the significance of our proposed AOs. However, the proposed approach is the most reliable approach because it is based on the MSM. In IFFWMSM operator the information is aggregated with the help of the MSM operator by using the Frank operations where some parameters are evolved. Hence, the IFFWMSM operator is the most appropriate operator to solve the MAGDM problems due to its interesting elasticity due to the involvement of the parameter.

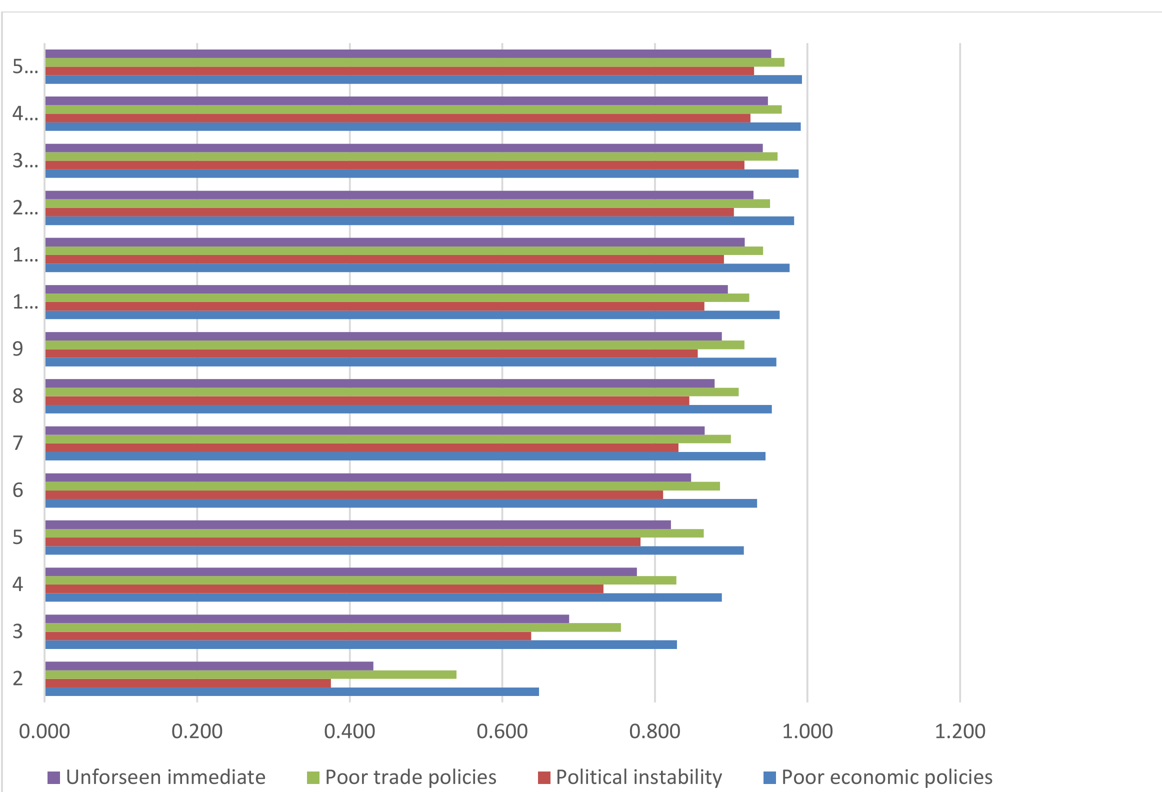

Fig. 1 Graph of changes in score values on different values of the parameter $\aleph$ from Table 7.

Table 8 Comparison of IFFWMSM operators with pre-existing operators.

Operator	Ranking of score values	Preference Orders
IFFWMSM	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
IFMSM [32]	$sc(\partial_1) > sc(\partial_2) > sc(\partial_4) > sc(\partial_3)$	$\partial_1 \succ \partial_2 \succ \partial_4 \succ \partial_3$
IFGW [40]	$sc(\partial_1) > sc(\partial_4) > sc(\partial_3) > sc(\partial_2)$	$\partial_1 \succ \partial_4 \succ \partial_3 \succ \partial_2$
IFFWA [44]	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$
IFAAWA [34]	$sc(\partial_1) > sc(\partial_3) > sc(\partial_4) > sc(\partial_2)$	$\partial_1 \succ \partial_3 \succ \partial_4 \succ \partial_2$

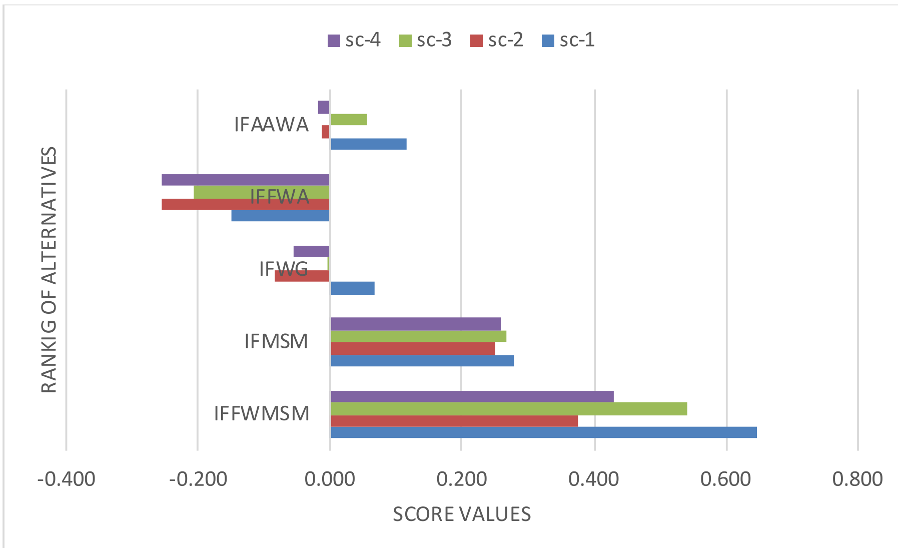

Fig. 2 Graph of the comparison between the score values obtained by different operators.

In the following, Table 8 shows the ranking order of risk factors obtained by different operator.

In Fig. 2, the score values obtained by the different aggregation operators tabulated in Table 8 are plotted. Fig. 2 shows the difference between the score values of these operators. In this figure, the score values obtained by different operators are plotted on the vertical axis. The lines in the graph shows same ranking of choices (risk factors) which shows the significance of proposed the operators.

6. Conclusion

In this manuscript, the MSM operator has been extended to the IFFMSM and IFFWMSM operators for the framework of IFS based on the FTNM and FTCNM. The basic properties of these operators have been observed and proved. Then the proposed operator has been applied to a real-life problem by considering a numerical example. In the example, the risk factors for any construction industry are examined with the help of the proposed operator that has provided in the form of the IFVs. Four risk factors are discussed and scored based on the proposed research. Finally, the results obtained by the proposed approach have been compared with the pre-existing aggregation operators i.e., IFFWA, IFWG, and IFMSM operators. Some important points have been stated below.

The proposed IFFWMSM operator is based on the FTNM and FTCNM operations and due to the involved parameter $\aleph$ the IFFWMSM becomes flexible. With the help of the proposed research, the risk factors for the construction industry are examined. Likely, we can examine risk factors for any other industry or company we want and can make the necessary steps to accommodate those risk factors that swear for the industry in the future. In short, we can evaluate and intimate the risk factors and take precautions. In the future, we aim to generalize the study to the framework defined in the [21 and 47], and [29]. We also aim to extend the developed model for the frameworks utilized in [21] and [40].

Ethical approval “This article does not contain any studies with human participants or animals performed by any of the authors”.

Data Availability

“Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study”.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] K.-P. Adlassnig, Fuzzy set theory in medical diagnosis, IEEE Trans. Syst., Man, Cybernet. 16 (2) (1986) 260–265.

[2] M. Akram, K. Ullah, D. Pamucar, Performance Evaluation of Solar Energy Cells Using the Interval-Valued T-Spherical Fuzzy Bonferroni Mean Operators, Energies 15 (1) (2022) 292.

[3] J.C.R. Alcantud, A.Z. Khameneh, A. Kilicman, Aggregation of infinite chains of intuitionistic fuzzy sets and their application to choices with temporal intuitionistic fuzzy information, Inf. Sci. 514 (2020) 106–117, https://doi.org/10.1016/j.ins.2019.12.008.

[4] R. Al-Husban, A.R. Salleh, A.G.B. Ahmad, Complex intuitionistic fuzzy normal subgroup, Int. J. Pure Appl. Math. 115 (3) (2017) 455–466.

[5] M. Ali, D.E. Tamir, N.D. Rishe, A. Kandel, Complex intuitionistic fuzzy classes, in: *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2027–2034.

[6] Z. Ali, T. Mahmood, K. Ullah, D. Pamucar, G. Cirovic, Power Aggregation Operators Based on t-Norm and t-Conorm under the Complex Intuitionistic Fuzzy Soft Settings and Their Application in Multi-Attribute Decision Making, Symmetry 13 (11) (2021) 1986.

[8] N. Alshehri, M. Akram, Intuitionistic fuzzy planar graphs. *Discrete Dynamics in Nature and Society*, *2014*.

[9] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Set. Syst. 20 (1) (1986) 87–96.

[10] X. Chen, C. Suo, Y. Li, Distance measures on intuitionistic hesitant fuzzy set and its application in decision-making, Comput. Appl. Math. 40 (3) (2021) 1–21.

[11] C.-H. Chu, K.-C. Hung, P. Julian, A complete pattern recognition approach under Atanassov's intuitionistic fuzzy sets, Knowl.-Based Syst. 66 (2014) 36–45.

[12] S.K. De, R. Biswas, A.R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis, Fuzzy Set. Syst. 117 (2) (2001) 209–213.

[13] G. Deschrijver, E.E. Kerre, A generalization of operators on intuitionistic fuzzy sets using triangular norms and conorms, Notes IFS 8 (1) (2002) 19–27.

[14] H. Garg, R. Arora, Novel scaled prioritized intuitionistic fuzzy soft interaction averaging aggregation operators and their application to multi criteria decision making, Eng. Appl. Artif. Intel. 71 (2018) 100–112.

[15] H. Garg, K. Kumar, Some Aggregation Operators for Linguistic Intuitionistic Fuzzy Set and its Application to Group Decision-Making Process Using the Set Pair Analysis, Arab. J. Sci. Eng. 43 (6) (2018) 3213–3227, https://doi.org/10.1007/s13369-017-2986-0.

[16] H. Garg, D. Rani, Some generalized complex intuitionistic fuzzy aggregation operators and their application to multicriteria decision-making process, Arab. J. Sci. Eng. 44 (3) (2019) 2679–2698.

[17] D.H. Hong, C.-H. Choi, Multicriteria fuzzy decision-making problems based on vague set theory, Fuzzy Set. Syst. 114 (1) (2000) 103–113.

[18] J.-Y. Huang, Intuitionistic fuzzy Hamacher aggregation operators and their application to multiple attribute decision making, J. Intell. Fuzzy Syst. 27 (1) (2014) 505–513, https://doi.org/10.3233/IFS-131019.

[19] A. Hussain, K. Ullah, H. Wang, M. Bari, Assessment of the Business Proposals Using Frank Aggregation Operators Based on Interval-Valued T-Spherical Fuzzy Information, J. Function Spaces 2022 (2022) e2880340.

[20] M.J. Khan, P. Kumam, W. Kumam, Theoretical justifications for the empirically successful VIKOR approach to multi-criteria decision making, Soft. Comput. 25 (12) (2021) 7761–7767.

[21] P. Liu, R. Dang, P. Wang, X. Wu, Unit consensus cost-based approach for group decision-making with incomplete probabilistic linguistic preference relations, Inf. Sci. 624 (2023) 849–880, https://doi.org/10.1016/j.ins.2022.12.114.

[22] P. Liu, Y. Li, P. Wang, Opinion Dynamics and Minimum Adjustment-Driven Consensus Model for Multi-Criteria Large-Scale Group Decision Making Under a Novel Social Trust Propagation Mechanism, IEEE Trans. Fuzzy Syst. 31 (1) (2023) 307–321, https://doi.org/10.1109/TFUZZ.2022.3186172.

[23] P. Liu, P. Wang, W. Pedrycz, Consistency- and Consensus-Based Group Decision-Making Method With Incomplete Probabilistic Linguistic Preference Relations, IEEE Trans. Fuzzy Syst. 29 (9) (2021) 2565–2579, https://doi.org/10.1109/TFUZZ.2020.3003501.

[24] P. Liu, K. Zhang, P. Wang, F. Wang, A clustering- and maximum consensus-based model for social network large-scale group decision making with linguistic distribution, Inf. Sci. 602 (2022) 269–297, https://doi.org/10.1016/j.ins.2022.04.038.

[25] Z. Liu, F. Teng, P. Liu, Q. Ge, Interval-Valued Intuitionistic Fuzzy Power Maclaurin Symmetric Mean Aggregation Operators and Their Application To Multiple Attribute Group Decision-Making, Int. J. Uncertain. Quantif. 8 (3) (2018) 211–232.

[26] Z. Liu, P. Liu, X. Liang, Multiple attribute decision-making method for dealing with heterogeneous relationship among attributes and unknown attribute weight information under q-rung orthopair fuzzy environment, Int. J. Intell. Syst. 33 (9) (2018) 1900–1928.

[27] J. Lu, C. Wei, J. Wu, G. Wei, TOPSIS method for probabilistic linguistic MAGDM with entropy weight and its application to supplier selection of new agricultural machinery products, Entropy 21 (10) (2019) 953.

[28] C. Maclaurin, A second letter to Martin Folkes, Esq.; concerning the roots of equations, with demonstration of other rules of algebra, Philos. Trans. R. Soc. Lond. Ser. A 1729 (36) (1729) 59–96.

[29] T. Mahmood, A novel approach towards bipolar soft sets and their applications, *J. Math.*, *2020*.

[30] R. Mesiar, Nearly Frank t-norms, Tatra Mt. Math. Publ 16 (127) (1999) 127–134.

[31] D. Pamučar, I. Badi, K. Sanja, R. Obradović, A Novel Approach for the Selection of Power-Generation Technology Using a Linguistic Neutrosophic CODAS Method: A Case Study in Libya, Energies 11 (9) (2018) Article 9, https://doi.org/10.3390/en11092489.

[32] J. Qin, X. Liu, An approach to intuitionistic fuzzy multiple attribute decision making based on Maclaurin symmetric mean operators, J. Intell. Fuzzy Syst. 27 (5) (2014) 2177–2190, https://doi.org/10.3233/IFS-141182.

[33] M. Seikh, U. Mandal, Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making, Granular Computing 6 (2021), https://doi.org/10.1007/s41066-019-00209-y.

[34] T. Senapati, G. Chen, R.R. Yager, Aczel-Alsina aggregation operators and their application to intuitionistic fuzzy multiple attribute decision making, Int. J. Intell. Syst. 37 (2) (2022) 1529–1551, https://doi.org/10.1002/int.22684.

[35] K. Ullah, Picture fuzzy maclaurin symmetric mean operators and their applications in solving multiattribute decision-making problems. *Math. Probl. Eng.*, *2021*.

[36] K. Ullah, N. Hassan, T. Mahmood, N. Jan, M. Hassan, Evaluation of investment policy based on multi-attribute decision-making using interval valued T-spherical fuzzy aggregation operators, Symmetry 11 (3) (2019) 357.

[37] P. Wang, P. Liu, F. Chiclana, Multi-stage consistency optimization algorithm for decision making with incomplete probabilistic linguistic preference relation, Inf. Sci. 556 (2021) 361–388, https://doi.org/10.1016/j.ins.2020.10.004.

[38] W. Wang, X. Liu, Intuitionistic fuzzy information aggregation using Einstein operations, IEEE Trans. Fuzzy Syst. 20 (5) (2012) 923–938.
[39] G. Wei, H. Garg, H. Gao, C. Wei, Interval-valued Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making, IEEE Access 6 (2018) 67866–67884.
[40] Z. Xu, R.R. Yager, Some geometric aggregation operators based on intuitionistic fuzzy sets, Int. J. Gen Syst 35 (4) (2006) 417–433.
[41] Z. Xu, R.R. Yager, Intuitionistic Fuzzy Bonferroni Means, IEEE Trans. Syst., Manand Cybernet., Part B (Cybernetics) 41 (2) (2011) 568–578, https://doi.org/10.1109/TSMCB.2010.2072918.
[42] D. Yu, Intuitionistic fuzzy geometric Heronian mean aggregation operators, Appl. Soft Comput. 13 (2) (2013) 1235–1246.
[43] L.A. Zadeh, Inf. Control*Fuzzy Sets* 8 (3) (1965) 338–353.
[44] X. Zhang, P. Liu, Y. Wang, Multiple attribute group decision making methods based on intuitionistic fuzzy frank power aggregation operators, J. Intell. Fuzzy Syst. 29 (5) (2015) 2235–2246, https://doi.org/10.3233/IFS-151699.
[45] Z. Zhang, Interval-valued intuitionistic fuzzy Frank aggregation operators and their applications to multiple attribute group decision making, Neural Comput. Applic. 28 (2017), https://doi.org/10.1007/s00521-015-2143-1.
[46] M. Zhao, G. Wei, C. Wei, J. Wu, Improved TODIM method for intuitionistic fuzzy MAGDM based on cumulative prospect theory and its application on stock investment selection, Int. J. Mach. Learn. Cybern. 12 (3) (2021) 891–901.
[47] X. Zhao, G. Wei, Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making, Knowl.-Based Syst. 37 (2013) 472–479, https://doi.org/10.1016/j.knosys.2012.09.006.

Further Reading

[7] Z. Ali, T. Mahmood, M.-S. Yang, TOPSIS Method Based on Complex Spherical Fuzzy Sets with Bonferroni Mean Operators, Mathematics 8 (10) (2020) Article 10, https://doi.org/10.3390/math8101739.