Efficiency of Elementary Schools in Belgrade: A DEA and SHAP Analysis

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Abstract: This study investigates the efficiency of elementary schools in the Belgrade region in preparing students for middle school admission, highlighting the critical role of socio-economic factors. Using a modified output-oriented Data Envelopment Analysis model with municipality score constraints, we assess school performance by comparing grade point averages and student numbers (inputs) to standardized test scores (outputs). Our findings reveal stark differences in efficiency across municipalities, with the highest-performing areas, such as Stari Grad, Voždovac, and Zemun, benefiting from dynamic economies and diverse opportunities. In contrast, outer-circle municipalities like Sopot, Mladenovac, and Obrenovac face challenges tied to limited economic prospects. Leveraging Shapley Additive Explanations analysis, we uncover that unemployment is the most influential factor, with municipalities with lower unemployment achieving significantly lower efficiency scores. These findings not only demonstrate the interconnectedness of socio-economic conditions and education but also emphasize the need for targeted interventions to bridge performance gaps. By addressing unemployment and fostering economic growth, particularly in disadvantaged areas, we believe that policymakers can create a more equitable educational landscape.

Keywords: Data Envelopment Analysis; Shapley Additive Explanations; Educational Efficiency

1. Introduction

An important issue in education is understanding why some elementary schools consistently outperform others in preparing students for middle school admission. This disparity often stems from socio-economic factors, such as unemployment, economic opportunities, and access to resources, which influence both student performance and school efficiency (Ozturk, 2008; Hayat et al., 2022). In Serbia, this disparity is particularly evident, with schools in socio-economically advantaged areas achieving better outcomes compared to those in disadvantaged municipalities (Pešikan & Ivić, 2021).

Understanding the efficiency of elementary schools is critical because it directly impacts students' academic success and their future opportunities. By identifying the factors that influence efficiency, we can uncover actionable insights to reduce educational disparities, improve resource allocation, and enhance overall system performance, ultimately fostering a better education system. In the educational domain, Data Envelopment Analysis has been widely used as a powerful tool to evaluate school efficiency by comparing inputs, such as student performance metrics, to outputs, such as standardized test scores. For example, in the higher education one can find ranking of different education systems in (Sun et al., 2023), or (Lee & Johnes, 2022) who evaluated teaching efficiency to graduate employment. More often, data envelopment analysis was used for ranking of schools (Henriques et al., 2022; Chiariello et al., 2022), but also for discovering determinants of success (Kounetas et al., 2023). Applying DEA in the context of education allows for a structured assessment of how schools convert resources and academic preparation into successful outcomes, providing valuable benchmarks for policymakers to identify inefficiencies and drive improvements in the education system. By identifying the factors that influence efficiency, we can uncover actionable insights to reduce educational disparities, improve resource allocation, and enhance overall education system performance. Studying education efficiency is not only academically intriguing but also socially impactful, as it helps identify what factors can hinder student success.

In Serbia, admission to middle schools, including gymnasiums and vocational schools, is determined by a standardized entrance exam known as the Final Exam, typically taken at the end of elementary education (8th grade). This exam assesses students' knowledge and skills in key subjects: Serbian language and literature (or the mother tongue), mathematics, and a third test which is selected in advance by the student and can be one of the following: history, geography, biology, physics, and chemistry. The results of the Final Exam are combined with students' grade point averages from the last three years of elementary education (6th, 7th, and 8th) to calculate their total score, which determines their eligibility for specific middle schools. Students rank their preferred schools and programs, and admission is competitive, with higher-ranking schools typically requiring better results due to limited capacity. The process ensures a merit-based transition from elementary to secondary education.

The key components of this approach include applying a data envelopment analysis model to measure the efficiency of elementary schools in transforming inputs, such as grade point averages and the number of students, into outputs, including standardized test scores. However, data envelopment analysis mathematical model should be adapted for the specificities of the educational system in Serbia. More specifically, the model is adapted so the efficiency scores are bounded for the average municipality efficiency scores are not overly high, nor low. This is one of the technical contributions of the paper. In addition to the data envelopment analysis, we wanted to inspect disparities in efficiency scores between municipalities by incorporating socio-economic factors like unemployment rates, average salaries, and active companies. To do so, we incorporate SHAP explanations to analyse the influence of socio-economic factors on municipal efficiency scores.

2. Methodology

Our research goal is to evaluate efficiency of schools regarding the admission to middle school and assess if there are differences in results that can be attributed to the socio-economic factors of the municipality.

The motivation for this analysis lies in understanding the efficiency of elementary schools in preparing students for this critical transition. Higher GPAs in 6th, 7th, and 8th grades are expected to correlate positively with higher Final Exam scores, as they reflect consistent academic preparation and mastery of curriculum content. Therefore, efficient schools should be those that maximize exam scores relative to their performance of students during the elementary education, and *vice-versa* inefficient schools should be those where exam scores are low compared to the grade average during the elementary education. The reason why we would like to assess socio-economic factors comes from the OECD's PISA 2022 assessment that indicates that socio-economically advantaged students (the top 25% in terms of socio-economic status) outperformed disadvantaged students (the bottom 25%) by a considerable margin (Organisation for Economic Co-operation and Development, 2022) in Serbia.

As a suitable methodology for efficiency analysis (Ji & Lee, 2010), we employ data envelopment analysis to assess the efficiency of each individual elementary school and inspect if there are differences in average scores within and between municipalities. This analysis aligns closely with the structure of Serbia's education system, where the transition to middle schools heavily depends on performance in the Final Exam, which includes the same subject areas as the outputs in the DEA model.

2.1 Design Choices

Data. To assess the efficacy of elementary schools, we utilized publicly available results from the reporting portal "My Middle School[21]," which represents the systematic and comprehensive collection of *ad-hoc* reports related to the school level performance, academic achievement, and standardized admission exam results.

[21] Web address: https://izvestavanje.mojasrednjaskola.gov.rs/

For the purposes of the research, we focused solely on the region of the capital city – Belgrade Region – and regular elementary schools. In other words, we removed schools aimed for the adult education, special needs education, and those for the education of the gifted. The former are schools aimed for students with development delays or those who failed to finish elementary school regularly due to various factors. These students often have lower academic performance, as teaching and grade evaluations take place after regular working hours. Upon completing elementary education, these individuals can continue their education by enrolling in secondary schools, including gymnasiums and vocational schools. However, the Adult Education Survey conducted in 2022 indicates that only 19.9% of adults engaged in formal or non-formal education or training, which is significantly below the European Union average of 45.1% (European Commission, 2024). It would be unfair to compare schools aimed at adult education or special needs education with regular schools because their objectives, target demographics, and evaluation metrics significantly differ. Their primary goals often centre on skill acquisition, individual progress, and reintegration into formal education or the workforce, rather than traditional academic achievement metrics.

On the other hand, schools for gifted students are designed to provide specialized, advanced instruction tailored to the exceptional abilities of their students in areas such as mathematics, science, or the arts. It is offered for students from the sixth grade of the elementary school and as such take only the most advanced students in the generation. These institutions often have access to additional resources, smaller class sizes, and customized learning environments that foster accelerated learning and innovative thinking. As a result, their performance metrics, such as academic achievement or standardized test scores, are naturally higher due to the exceptional aptitude of their student body. Including these schools in a comparison with regular elementary schools, which serve a broader and more diverse range of abilities, would create an uneven playing field and misrepresent the efficiency of regular schools by failing to account for these inherent advantages.

As a result, we obtained 176 elementary schools in Belgrade regions distributed over 17 municipalities of the Belgrade region.

Selection of input and output attributes. To assess the efficiency of the elementary school we selected the number of students and grade point averages at 6th, 7th, and 8th grade as inputs and average scores on mother tongue, mathematics, and the third test as outputs is highly relevant to the middle school admission process. In an ideal scenario, efficiency would depend solely on the selected inputs, reflecting the schools' ability to objectively assess students' knowledge throughout the elementary education and consequently translate knowledge into desired academic outcomes. However, we recognize that other external factors, such as socio-economic conditions, may also significantly influence school performance. That effects can be in a form of an exogenous influence (socio-economic factors influence inputs only), unobserved heterogeneity in the outcome (socio-economic factors influence outputs only), or even a confounder effect (socio-economic factors influence both inputs and outputs). To have a notion of account for these effects, we aim to explore how socio-economic factors could explain the observed differences in efficiency scores across schools. This approach will provide a more comprehensive understanding of the disparities in school efficiency and help identify areas for targeted interventions.

2.2 Data Envelopment Analysis Model

To calculate the efficiency of schools we adopted an output-oriented data envelopment analysis model with several adjustments. Since schools belong to a municipality who do have some notion of influence on the school organization and functioning through financial and organizational benefits, we want to ensure that average efficiency does not differ to much from other municipalities. Therefore, when observing the efficiency scores, we would like every municipality to have at most γ percentage difference of average efficiency scores compared to the remaining schools. One way to ensure that is to have a multi-step procedure where we would calculate the efficiency scores, observe municipality average efficiency scores, construct a constraint and repeat the data envelopment analysis with new set of constraints. In case of an infeasible solution, we would relax the constraints and try again. A more elegant solution would be to incorporate a set of constraints in the mathematical model directly. To do that, we must introduce one mathematical model for calculation of efficiencies for every school instead of n mathematical models for each school separately, which is common in data envelopment analysis (Camanho et al., 2024).

We use the fact that the sum of linear functions is also a linear function (Aggarwal et al, 2020). Therefore, solving for n models can be posed as a single model. More importantly, the optimal solution of such formulation will be the same as the sum of n individual models. Consequently, efficiency scores for each school will remain the same with a caveat that this kind of formulation can result in a large number of variables one needs to optimize for, as well as in a large number of constraints. Effects of the model increase can be found in (Radovanović et al., 2022).

The model we employed is presented below:

$$\max \sum_{d=1}^{n} \sum_{r=1}^{s} u_{rd} y_{rd} \tag{1}$$

$$s.t.$$

$$\sum_{i=1}^{m} v_{id} x_{id} = 1, d = 1, \dots, n \tag{2}$$

$$\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0, \forall d, j = 1, \dots, n \tag{3}$$

$$\frac{1}{N_c} \sum_{d \in c} \sum_{r=1}^{s} u_{rd} y_{rd} - \frac{1}{N_{\rceil c}} \sum_{d \notin c} \sum_{r=1}^{s} u_{rd} y_{rd} \leq \gamma \tag{4}$$

$$\frac{1}{N_{\rceil c}} \sum_{d \notin c} \sum_{r=1}^{s} u_{rd} y_{rd} - \frac{1}{N_c} \sum_{d \in c} \sum_{r=1}^{s} u_{rd} y_{rd} \leq \gamma \tag{5}$$

$$v_{id} \geq \varepsilon, i = 1, \dots, m \ \wedge d = 1, \dots, n \tag{6}$$

$$u_{rd} \geq \varepsilon, r = 1, \dots, s \ \wedge d = 1, \dots, n \tag{7}$$

This DEA model optimizes the efficiency of decision-making units, in this case elementary school, while incorporating constraints to ensure fairness across groups or categories (in this case municipalities). Since the model is output-oriented one seeks to maximize the weighted sum of outputs presented in equation (1) ($u_{rd} y_{rd}$) for each school d, where u_{rd} represents the weights for outputs r, and y_{rd} is the output values. To ensure that efficiency can be interpreted as an efficiency (Ahn et al., 1988) the weighted sum of inputs ($v_{id} x_{id}$) for each school d must equal 1 (equation (2)), where v_{id} represents the weights for inputs i, and x_{id} is the input values. In addition, to ensure that no school can dominate others in terms of efficiency we must introduce a set of constraints presented in equation (3). For every school d, the weighted sum of outputs minus the weighted sum of inputs for all other schools must be less than or equal to 0. One can interpret this set of constraints also that efficiency can be at most one. In the context of school efficiency, every school is compared to every school in the Belgrade region to form an absolute frontier.

One of the contributions of this paper is introduction of statistical parity set of constraints presented in equations (4) and (5). These constraints enforce that the average efficiency of schools within a specific municipality c should not significantly deviate from those outside the municipality by a factor γ. More specifically, equation 4 ensures the average efficiency within the category does not exceed that of the rest (schools not in the municipality c) by more than a threshold (γ), while equation 5 ensures the reverse, maintaining fairness and comparability.

Finally, to ensure faster convergence of optimization procedure we normalized the data using L_∞

norm (Aggarwal et al, 2020). Also, we set γ to 0.1 and ε to 0.01 but also bounded the maximum of both u_{rd} and v_{id} so the contribution of a single or several outputs or inputs is limited. For the optimization of the model we used interior point method.

3. Results and Discussion

After conduction the data envelopment analysis, we have obtained that only two elementary schools were indeed efficient. The histogram presented in Figure 1 shows efficiency scores are ranging from 0.5959 to 1, with majority of the schools achieved relatively high efficiency scores (0.75–0.90).

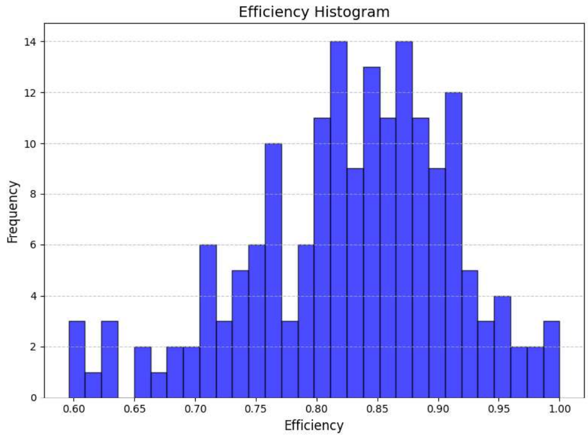

Figure 13: Efficiency scores histogram

If we are to compare the results on the test and efficiency, we can notice that higher test scores consistently correspond to greater efficiency (Figure 2), with strong clustering of high-efficiency observations in the upper-right quadrants of all plots. Similarly, there is a slight variation in test scores which correspond to low efficiency in the lower-left quadrants.

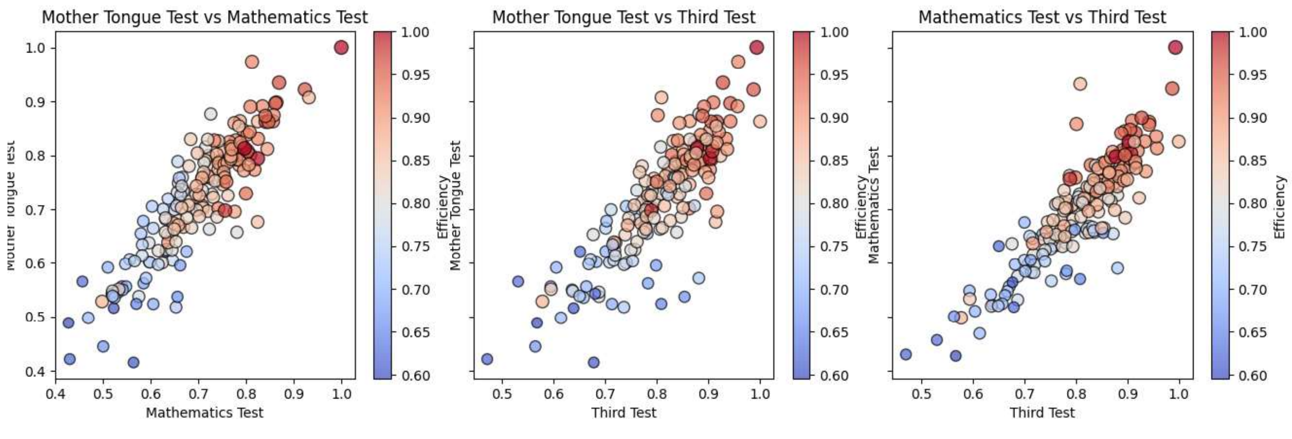

Figure 14: Correlation between efficiency scores and output

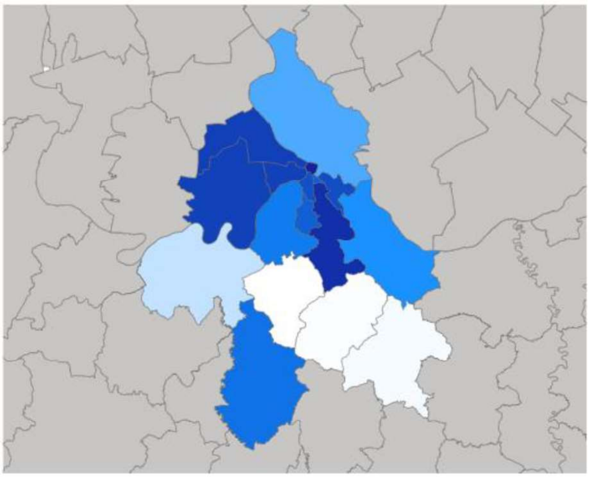

Figure 15: Average efficiency scores across municipalities

We can observe in Figure 3 that efficiency scores differ across municipalities. White represents lower efficiency scores, while darker blue color represents higher efficiency scores. Based on the average efficiency scores obtained, the top three municipalities are Stari Grad (0.8816), Voždovac (0.8722), and Zemun (0.8610). These municipalities more-or-less belong to the city center. Stari Grad is a hub for commerce, tourism, and culture, contributing to its high economic activity. Voždovac hosts significant industrial zones and residential neighbourhoods, supporting a diverse economy, and Zemun, known for its developed industries, boasts sectors ranging from manufacturing to services, enhancing its economic standing.

On the other side of the efficiency spectrum are Sopot (0.7304), Mladenovac (0.7321), and Obrenovac (0.7487). Sopot and Mladenovac are more rural, with economies primarily based on agriculture and small-scale industries, which may limit their economic opportunities.

To compare the efficiency scores with the socio-economic factors we need to use data from the Republic Statistical Office Overview of the state and development of municipalities[22] database for 2023. More specifically, we selected several socio-economic factors to explain efficiency scores as they capture key aspects of economic activity (e.g., active companies, entrepreneurs, and salaries), population demographics (e.g., ageing index and population estimates), and social welfare (e.g., child allowances and institutional care). Additionally, employment and unemployment metrics provide insights into labour market dynamics. Namely, alphabetically we selected Active companies per 1,000 inhabitants, Active entrepreneurs per 1,000 inhabitants, Ageing index, Number of beneficiaries of increased allowance for assistance and care of another person per 1,000 inhabitants, Number of beneficiaries of increased child allowance (aged 0-17) per 1,000 inhabitants, Number of beneficiaries of public residential institutions aged 65 and over per 1,000 inhabitants, Percentage of registered employees by municipalities of residence, and Registered unemployed per 1,000 inhabitants.

To inspect the association between efficiency scores and socio-economic attributes we employ Shapley value explanations (Lundberg, 2017). It is a model-agnostic technique based on cooperative game theory that explains the contribution of each feature to a model's predictions. It calculates the average marginal effect of a feature across all possible feature combinations. Global summary plot is presented in Figure 4.

[22] Web address: http://devinfo.stat.gov.rs/Opstine/libraries/aspx/home.aspx

Figure 16: SHAP Summary plot

The SHAP summary plot reveals that *Registered unemployed per 1,000 inhabitants* is the most influential factor, with higher unemployment significantly impacting the efficiency score. More specifically, municipalities with lower unemployment are performing better than municipalities with higher unemployment. An interpretation can be found in (Ananat et al., 2011) who concluded that living in a family with unemployed parents leads to increased stress and decreased well-being of children, which can adversely influence children's educational outcomes. What is more interesting is that unemployment is more important than employment (attribute *Registered employees by municipalities of residence comparing to population number*) as SHAP values are higher in intensity. Another interpretation is given by (Bordot, 2022), which puts unemployment into perspective of lack of opportunities of work in the vicinity of residence. Given that municipalities with higher unemployment in the Belgrade region are those at outer circle of Belgrade, this might suggest that lack of economic opportunities leads to lower education performance. This might be the case as other important factors by SHAP values are *Active companies* and *Average annual net salaries and wages*.

Features like child allowances, ageing index, and institutional care have smaller impacts, suggesting less direct influence on the outcome. Research indicates that socio-economic factors such as child allowances, ageing index, and institutional care have varying impacts on educational outcomes. As (Baker et al., 2024) stated, increased child benefits can improve children's mental health but may not significantly affect standardized test scores. The main reason for such results is that mental health is a prerequisite for good performance, but student must have capacity to reach the better score.

4. Conclusions

This study highlights the significant role of socio-economic factors in shaping the efficiency of elementary schools in the Belgrade region. Through an innovative data envelopment analysis model and SHAP analysis, we reveal that unemployment is the most influential factor, with municipalities with lower unemployment achieving lower efficiency. Economic drivers such as active companies and average salaries further underscore the connection between local economic conditions and educational outcomes. While factors like child allowances and institutional care have less direct impact, they remain important for addressing broader inequalities in efficiency scores. Another benefit one can have from the data envelopment analysis is the set of role models, or efficient schools that utilized the resources better. This can be used as a part of a discussion for the policy and decision making.

The sole fact that results highlight unemployment as a factor that significantly impacts educational outcomes, this suggests the need for targeted interventions in high-unemployment municipalities. Investing in mental health programs, tutoring, and skill-building initiatives can mitigate the adverse effects on students. Additionally, addressing regional disparities in economic opportunities through job creation and infrastructure improvements can enhance access to education and reduce inequality. Promoting holistic policies that integrate economic and educational support can improve overall student performance while fostering long-term social equity and workforce development.

In the future work, we plan to extend the DEA efficiency scores to the entire Serbia, not just Belgrade Region. In addition, we plan to implement preference elicitation methods that would guide the efficiency frontier for different region, municipalities, and school types. Preference elicitation would define input-output combinations that would consequently define groups of similar elementary schools that could be an input to DEA method or, more preferably, a *human-in-the-loop* procedure for DEA method. That way, a decision- or policy-maker would see the consequences of choices and correct the efficiency score calculation in real-time. Another line of research would be aimed at replacing linear input and output utility function DEA assumes with more complex utility functions. One approach worth considering is replacing the utility function with Gaussian processes or neural networks with autoencoder structures.

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