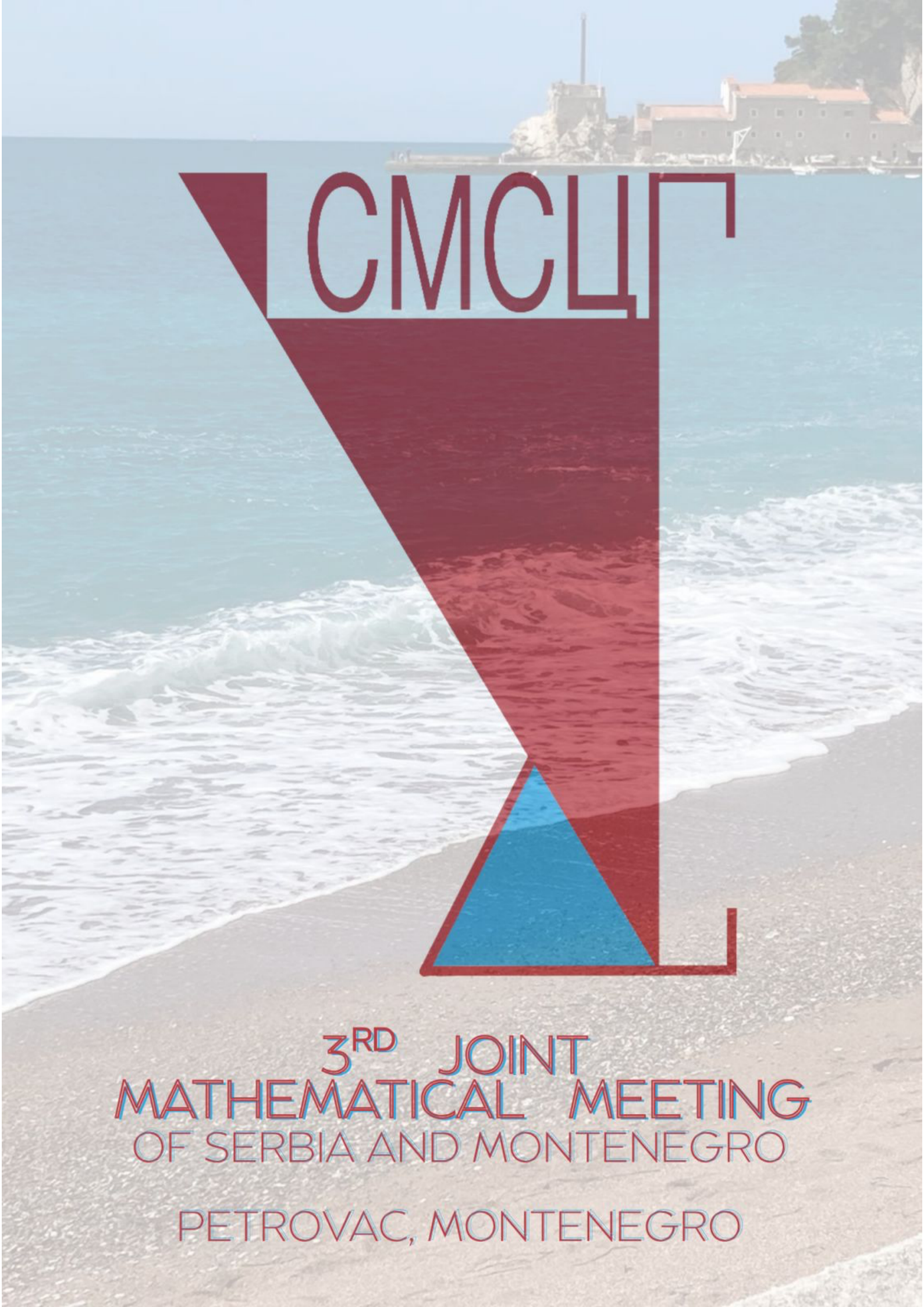

3RD JOINT MATHEMATICAL MEETING OF SERBIA AND MONTENEGRO

PETROVAC, MONTENEGRO

3rd JOINT MATHEMATICAL MEETING OF SERBIA AND MONTENEGRO

Petrovac, 2 -5 October 2025

Welcome Address

It is our great pleasure to welcome you to the 3rd Joint Mathematical Meeting of Serbia and Montenegro (SMSCG 2025), which will take place in the beautiful coastal town of Petrovac, Montenegro, from October 2 to 5, 2025.

This meeting continues the tradition of bringing together mathematicians from Serbia, Montenegro, and abroad to share their research, strengthen professional ties, and foster new collaborations. The 2025 edition covers a broad spectrum of mathematical topics, ranging from probability and statistics, functional analysis, and differential equations, to algebra and geometry. We are especially pleased that one of the sections will be dedicated to our distinguished researcher Rade Živaljević.

For the first time, the conference will also feature several sessions devoted to mathematics education, which will bring together not only researchers but also a large number of teachers from across the region. We hope that the exchange of ideas will contribute to the advancement of mathematics teaching in the region.

We extend our sincere gratitude to all participants, speakers, and contributors for making this meeting possible. Special thanks go to the local organizing committee, supporting institutions, and sponsors, whose efforts and dedication ensure the success of this event.

We trust that the 3rd Meeting of Mathematicians of Serbia and Montenegro will be both scientifically enriching and personally inspiring, and that your stay in Petrovac will be a memorable experience.

Welcome to Petrovac, and to SMSCG 2025!

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Project "Mathematical analysis, optimization and machine learning" funded by Ministry of Education, Science and Innovation

Abstracts

Asymptotic results for some U- and V- statistics for model specification testing under missing data

Danijel G. Aleksić
Faculty of Organizational Sciences, University of Belgrade
Faculty of Mathematics, University of Belgrade

Marija Cuparić, Bojana Milošević
Faculty of Mathematics, University of Belgrade

Missing data issues are a common problem in practice. Although numerous model specification tests have been proposed for complete data, the literature addressing incomplete data remains sparse. Seeking to fill this gap, we focus on non-degenerate U- and V-statistics, with the ultimate goal of applying them in the context of independence testing via the well-known Kendall's tau statistic. We assume that the data are Missing Completely At Random (MCAR). For general U- and V-statistics, we present their limiting distributions under the complete-case approach. For the specific case of Kendall's tau, we provide additional results under median imputation and outline the path toward deriving analogous results for other commonly used imputation algorithms.

Mathematical games as a method of motivating students in mathematical education

Vojislav Andrić
Mathematical Club "Diophantus", Valjevo

One of the most significant challenges in education in general, and particularly in mathematics education, is the motivation of students. This issue is often addressed by using mathematical games as a teaching method, since games are an enjoyable activity inherent to human nature.

The aim of this report is to demonstrate through specific examples of mathematical games how a well-designed and purposeful game can be effective in mathematics education. Furthermore, it aims to present the modalities of games used in mathematics education, showcasing mathematical games that are characterized by individual student engagement, games conducted in pairs, and games with a group character. The report will also showcase games with a competitive nature, which adds a new and additional sense of motivation for students. Discussions will include the discovery of winning strategies (if they exist).

Special attention will be given to the principles of designing games in the context of achieving educational outcomes, and to methods of evaluating the effects of games

and gameplay. Often, mathematical games are used to make lessons more interesting while overlooking the educational impacts of the games.

Other aspects of using games will also be mentioned, such as the age of the students, the educational level of the games, and the correlation of educational subjects through play.

Normed spaces of pointwise Bochner and Dunford p-integrable operator valued functions

Miloš Arsenović, Mihailo Krstić, Stefan Milošević, Matija Milović
Faculty of Mathematics, University of Belgrade

We consider two scales of normed spaces, both wider than the space of Bochner integrable functions which take values in the space of bounded operators $\mathcal{B}(X,Y)$. Also, we introduce the pointwise Bochner integral $\int_E^{pB} \mathcal{A}_t \, d\mu(t) : X \to Y$ of a family of operators $(\mathcal{A}_t)_{t\in\Omega}$ and investigate some properties of the operator $\int_E^{pB} \mathcal{A}_t \, d\mu(t) : X \to Y$. Several examples and counterexamples will be given. We also discuss, in the Hilbert space setting, inequalities which involve order on the space of operators.

Polyominoes with maximal number of deep holes

Djordje Baralić
Mathematical Institute of the Serbian Academy of Sciences and Arts

Shiven Uppal
Sanskriti School, New Delhi

We are studying the maximum number of deep holes h_n that an n-omino can enclose, ensuring that the boundary of a hole is disjoint from the boundary of any other hole and from the outer boundary of the n-omino. Using versatile methods we establish the inequality

$$\frac{n}{3} - \frac{16}{9}\sqrt{3n+1} + o(\sqrt{n}) \;<\; h_n \;<\; \frac{n}{3} - \frac{1}{18}\sqrt{48n\pi + \pi^2 + 12\pi} + o(\sqrt{n}) \qquad (1)$$

and show that h_n has linear growth asymptotically, in fact

$$h_n = \frac{n}{3} + O(\sqrt{n}).$$

Nonparametric tests for independence in the presence of cured data

Marija Cuparić, Bojana Milošević
Faculty of Mathematics, University of Belgrade

We address the problem of testing independence between two randomly right-censored variables under the assumption that the data follow an additive mixture model with a cured fraction. Four classes of test statistics, inspired by Kendall's τ and its extensions, are proposed. We investigate their asymptotic behavior as well as their performance in finite samples. A comprehensive empirical study confirms the strong competitiveness of the proposed methods.

Testing independence in high dimensions: a Kendall's τ extension for incomplete data

Marija Cuparić, Bojana Milošević
Faculty of Mathematics, University of Belgrade

Jelena Radojević
Faculty of Civil Engineering, University of Belgrade
Faculty of Mathematics, University of Belgrade

We consider the problem of testing independence in high-dimensional settings with missing data. Building upon a recently proposed Kendall-based statistic, we introduce two new modifications specifically designed to accommodate incomplete observations. The proposed methods are studied from both theoretical and empirical perspectives. A comprehensive simulation study illustrates the robustness and applicability of the new approaches. The findings contribute to the development of nonparametric methods for analyzing high-dimensional and incomplete data structures.

A new approach of modelling dependence in random environment INAR processes

Teodora D. Čamagić, Aleksandar S. Nastić, Miodrag S. Djordjević
Faculty of Sciences and Mathematics, University of Niš

Discrete-time count processes observed in natural and social phenomena often exhibit complex dependency patterns that are not adequately captured by conventional stationary INAR models operating under a single random environment. In this paper, we introduce a new integer-valued autoregressive (INAR) model in a random environment that allows separate control of the process mean and autocorrelation structure through two distinct Markov processes. The first random environment influences the parameters of the marginal distribution, while the second directly determines the values of the thinning operator, thereby dynamically controlling the correlation between successive observations.

This structural innovation provides greater flexibility in modeling nonlinear and nonstationary dependency patterns while preserving the interpretability of the model. In the presentation, we will focus on an intuitive explanation of the model construction, including motivation, definition, and examples illustrating how different environmental regimes affect the process behavior. Additionally, a simulation study will be presented demonstrating the efficiency and dynamic nature of the proposed approach.

Some examples of Grothendieck group of ring

Dalibor Danilović
Faculty of Mathematics, University of Belgrade

Grothendieck group $K_0(R)$ is subtle invariant associated to every ring R, and is defined as group completion of Abelian monoid of finitely generated projective modules over R. In practise, it is often hard to calculate this group. In this talk we will give some interesting examples when it is possible to calculate group $K_0(R)$.

Orlik-Solomon algebra of $J_{n,2}$ arrangement

Marija Došljak
Faculty of Science and Mathematics, University of Montenegro

We present computation of the cohomology ring of the complement of hyperplane arrangement $J_{n,2}$. It is based on the description of the Orlik-Solomon (OS) algebra, which can be associated to an arrangement and encodes the topology of its complement via combinatorial data. After recalling the basic construction of the OS algebra, we establish a linear ordering of hyperplanes and characterize independent sets, circuits, and broken circuits that determine the algebra's relations.

For the $J_{n,2}$ arrangement of hyperplanes the analysis focuses on relations of different lengths among generators, derived from the affine geometry of the unit cube. Explicit results are obtained for relations of length two, three, four, and five, with precise combinatorial conditions under which these occur.

We then apply these general results to the case of $J_{4,2}$, which consists of eleven hyperplanes. The generators of its OS algebra and the complete list of relations of lengths two and three are presented in detail, showing how they follow from the obtained general statements.

These results provide a complete description of the cohomology ring of the complement of the hyperplane arrangement $J_{4,2}$.

The talk is based on the results of the joint work with Svjetlana Terzić.

Optimization approach to the problem of spectral reconstruction of graphs

Robert Doža
Faculty of Mathematics, University of Belgrade

Marko Milenković
École polytechnique fédérale de Lausanne - EPFL

Predrag Đorđević, Tatjana Davidović
Mathematical Institute of the Serbian Academy of Sciences and Arts

Milica Anđelić
Kuwait University

Optimization problems on graphs consists of finding graphs (or a family of graphs) that minimize or maximize a given invariant (or some combination of the considered invariants). These problems usually belong to the NP-hard class making the exhaustive enumeration of all possibilities a unacceptable approach. Efficient optimization algorithms developed for these problems can help theoreticians to make hypothesis and get some useful hints how to analytically prove them. We consider a spectral reconstruction of graphs (SRG), an open problem from the spectral graph theory. It is a known fact that graphs are not uniquely determined (characterized) by their spectra in the general case. This means that, for certain graphs, there could exist several cospectral mates, i.e., non-isomorphic graphs corresponding to the same spectrum. The objective of SRG is to find at least one graph with a spectrum equal to a given constant vector. This problem is very important and has numerous real-life applications. A common approach to perform incomplete search in SRG is to develop metaheuristic tailored for this particular problem. Metaheuristic methods are general-purpose, stochastic, incomplete search algorithms to address NP-hard optimization problems. We develop several approaches to SRG, based on Variable Neighborhood Search (VNS) metaheuristic, taking into account the graph properties defined by its spectrum. We evaluate the performance of the proposed algorithms on the reconstruction of some well-known graphs and their potential non-isomorphic cospectral mates. Besides providing a graph for a given spectrum, the proposed algorithms can find cospectral mates, indicating that the corresponding graphs are not characterized by the spectrum. However, if a single graph is found, we cannot conclude that it is unique for a given spectrum.

Isoperiodic deformations of meromorphic differentials on Riemann surfaces and applications to Mathematical Physics

Vladimir Dragović
The University of Texas at Dallas
Mathematical Institute of the Serbian Academy of Sciences and Arts

We study deformations of elliptic and hyperelliptic Riemann surfaces and of a Abelian differential of the second or third kind on the surface which preserve the periods of the differential with respect to a chosen canonical homology basis of the surface. We derive differential equations with rational coefficients governing the deformations. We apply these results to the algebra-geometric solutions to the Boussinesq equation, KdV equation, sine-Gordon equation, the Neumann system, the Toda lattice, and SU(N) Witten-Seiberg theory. The talk is based on new joint results with Vasilisa Shramchenko and:

[1] V. Dragović, V. Shramchenko, *Isoharmonic deformations and constrained Schlesinger systems*, arXiv: 2112.04110.

[2] V. Dragović, V. Shramchenko, *Deformation of the Zolotarev polynomials and Painlevé VI equations*, Letters in Mathematical Physics, 111, 75, 28 p. 2021.

[3] V. Dragović, V. Shramchenko, *Algebro-geometric approach to an Okamoto transformation, the Painlevé VI and Schlesinger equations*, Annales Henri Poincaré, Vol. 20, No. 4, 1121–1148, 2019.

Equivariant fiber Shape Category of Equivariant Continuous Maps

Maia Dzadzamia, Lela Turmanidze, Ruslan Tsinaridze
Department of Mathematics, Batumi Shota Rustaveli State University

The shape theory of continuous maps is a new branch of classical shape theory. It is a meaningful extension of the homotopy theory of absolute neighborhood retracts for the category of maps of metrizable spaces. In this report we investigate equivariant theory of absolute (neighborhood) retracts and extensors for maps of topological $\boldsymbol{G}$-spaces from a weakly hereditary class. Here we define the notion of equivariant fiber resolution of $\boldsymbol{G}$-map. Obtained results we use for construction the equivariant fiber shape category of $\boldsymbol{G}$-maps.

Exponential and logarithmic functions in Banach algebras: Fréchet derivatives and Taylor approximations

Bogdan D. Djordjević
Mathematical Institute of the Serbian Academy of Sciences and Arts

Dušan D. Djordjević
Faculty of Sciences and Mathematics, University of Niš

This talk concerns the behavior of the exponential and logarithmic functions defined in unital Banach algebras: we will demonstrate their approximative properties and amenability with the Taylor polynomial. Accordingly, we will provide recursion-free formulae for the respective Fréchet derivatives of arbitrary orders.

C^*- valued regularly varying functions

Dragan S. Djordjević
Faculty of Sciences and Mathematics, University of Niš

We investigate integrals of C^*-algebra-valued functions or C^*-algebra-valued measures. Results are applied to algebra-valued regularly varying functions and algebra-valued orthogonal polynomials.

Existence and uniqueness theorems of approximate solutions of NSDEs

Dušan D. Djordjević, Miljana Jovanović, Marija Milošević
Faculty of Sciences and Mathematics, University of Niš

It is a common thing that some equations cannot be solved. On the other hand, in some cases, it can be shown that the equations have unique solutions. In those cases, some alternative approaches can be used, such as construction of an approximate solution. Here, besides the problem of existence and uniqueness (EU) of solution of an initial neutral stochastic differential equation with constant delay (NSDE), we are more concerned for the EU of an approximate solution. The approximate solution is obtained as a solution to a NSDE which is derived from the initial NSDE via Taylor expansions of its coefficients under some uncommon conditions, such as the polynomial conditions.

The use of technical terms in younger grades of elementary school and their impact on the development of empathy towards mathematics in students

Olivera V. Djordjević
Primary School "Njegoš", Niš

In this paper, we study the use of technical terminology in mathematics and its influence on the multi-layered intellectual-emotional development of students. We show the positive and negative aspects, prescribed by the curriculum for younger grades of elementary schools, on the creation of different levels of empathy in students towards mathematics.

$\mathbb{Z}_2$-homology of the orbit spaces $G_{n,2}/T^n$

Vladimir Ivanović, Svjetlana Terzić
Faculty of Science and Mathematics, University of Montenegro

We study the $\mathbb{Z}_2$-homology groups of the orbit space $X_n = G_{n,2}/T^n$ for the canonical action of the compact torus T^n on a complex Grassmann manifold $G_{n,2}$. Our starting point is the model (U_n, p_n) for X_n constructed by Buchstaber and Terzić (2022), where $U_n = \Delta_{n,2} \times \mathcal{F}_n$ for a hypersimplex $\Delta_{n,2}$ and a universal space of parameters $\mathcal{F}_n$ defined in the works of Buchstaber and Terzić (2019, 2022). It was proved by Buchstaber and Terzić (2023) that $\mathcal{F}_n$ is diffeomorphic to the moduli space $\mathcal{M}_{0,n}$ of stable n-pointed genus zero curves. We exploit the results of Keel (1992) and Ceyhan (2009) on the homology groups of $\mathcal{M}_{0,n}$ and express them in terms of the stratification of $\mathcal{F}_n$ incorporated in the model (U_n, p_n). As a result we provide an inductive, with respect to n, description of cycles in X_n. We also obtain explicit formulas for the $\mathbb{Z}_2$-homology groups of X_5 and X_6. The results for X_5 recover by a different method the results of Buchstaber and Terzić (2023) and Süss (2020). The results for X_6 seem to be new.

On the importance of the topic of complex numbers in secondary school mathematics teaching with implications on faculty teaching at the Faculty of Electrical Engeneering

Bratislav Iričanin
Faculty of Electrical Engineering, University of Belgrade
Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac

The presentation describes some important aspects in the teaching of mathematics at the high school level of education related to the topic of complex numbers. Apart from the direct importance in shaping the important mathematical (and not only such) notions, the direct consequences of the previous outcome are also analyzed to continue education at the higher education level. In particular, emphasis is placed on consideration case of teaching at the Faculty of Electrical Engineering, although teaching at other technical faculties could also be considered here.

The Cauchy-Leray transform and Λ-function on strongly $\mathbb{C}$-convex hypersurfaces

Luke Edholm
Faculty of Mathematics, University of Vienna

The Cauchy-Leray (or Leray) transform $\mathbf{L}$ is a higher-dimensional analogue of the planar Cauchy transform. It can be defined on $\mathbb{C}$-convex domains in n-dimensional space. When the boundaries of these domains have reasonable geometric regularity, $\mathbf{L}$ defines a skew projection onto the Hardy space of L^2-boundary values of holomorphic functions.

In this talk we introduce the Cauchy-Leray Λ-function, a projectively invariant function that measures the deviation of the Leray kernel and the Szego kernel, the reproducing Hilbert space kernel of the Hardy space. The boundary values of this function encode important analytic information about $\mathbf{L}$ and we show that these values are closed related to an invariant curvature function that can be defined on the boundaries of smoothly bounded strongly $\mathbb{C}$-convex domains. This relation has interesting implications for the mapping properties of $\mathbf{L}$ and closely related operators.

This work is joint with Dave Barrett.

Implicit Schwarz domain decomposition method for a Rayleigh-Benard problem

Henar Herrero, Darío Martínez and Francisco Pla
Universidad de Castilla-La Mancha

This work presents an implicit Schwarz domain decomposition method based on Legendre collocation method for a Rayleigh-Bénard problem. Legendre collocation is ill-conditioned. This strategy transforms the method into a well-conditioned one, so that even turbulence can be obtained. The convergence of the method has been proved theoretically. A numerical algorithm of the method is implemented, tested and compared with previous procedures. Errors and computational times are compared with the alternating Schwarz method. The method is more accurate because it avoids overlapping errors between domains and stagnation errors. The same order one of the time discretization is obtained. The computation times are shorter for an asymmetric number of subdomains.

Behold a theorem

Nebojša Ikodinović
Faculty of Mathematics, University of Belgrade

The title refers to the origin of the word theorem ($\theta\varepsilon\omega\rho\eta\mu\alpha$), derived from the Greek verb to look, to observe ($\theta\varepsilon\omega\rho\varepsilon\iota\nu$), which indicates that observation preceded proof in the birth of mathematics. At the same time, the title emphasises an important psychological phenomenon, the so-called *aha-moment* which strongly enhances both the cognitive and emotional components of learning. The main thesis of the presentation is that heuristically orientated teaching directly promotes the development of students' cognitive abilities and indirectly stimulates the growth of mathematics and its application in all areas of human activity. To defend this thesis, we will make connections between recent findings in cognitive science, the natural development of mathematical ideas, and the goals of education in today's society. All general considerations will be translated into concrete teaching examples related to the concept of similarity, which is poorly developed in school mathematics, especially in its early stages. Thales' measurement of the height of the pyramid will be considered within a typical school context, and then further analysed from the point of view of practical application (with attention to the reliability and scope of Thales' method) as well as within the context of the axiomatic foundations of Euclidean plane geometry (with an overview of the proofs of Thales' theorem). The starting point of this analysis will be a somewhat paradoxical twist: Thales' revolutionary method of measurement, with which true geometry was born, receives its justification only in the sixth book of Euclid's Elements, with which the foundation of plane geometry is completed. The aim of the analysis is to attempt to situate school mathematics between these two extremes. The main conclusions of the analysis will be summarized as didactic principles on which the organization of teaching should rest: 1. the principle of minimality; 2. the principle of adaptation; 3. the principle of activity.

The laureates of World Laureates Association Prize in Computer Science or Mathematics

Milojica Jaćimović
Montenegrin Academy of Sciences and Arts

Nevena Mijajlović, Ajlan Zajmović
Faculty of Science and Mathematics, University of Montenegro

The World Laureates Association Prize (WLA Prize) is an international science award established in Shanghai in 2021, initiated by the World Laureates Association. Each year, the WLA Prize is awarded in two categories: "WLA Prize in Computer Science or Mathematics" and "WLA Prize in Life Science or Medicine". The total award for each prize, which may be divided among up to four laureates, is RMB 10 million (approximately 1.4 million USD).

The laureates in Computer Science or Mathematics have included: Michael Jordan, University of California, Berkeley (2022); Arkadii Nemirovski, Georgia Institute of Technology, and Yurii Nesterov, Université catholique de Louvain (2023); and Jon Kleinberg, Cornell University (2023).

In our talk, we will briefly present their biographies and the achievements for which they were awarded. We will focus in more detail on the results in the field of convex optimization, including algorithm complexity, accelerated methods in convex optimization, and interior-point polynomial-time algorithms (Y. Nesterov, A. Nemirovski).

Mathematical foundations of machine learning in hyperbolic spaces

Vladimir Jaćimović
Faculty of Science and Mathematics, University of Montenegro

Learning low-dimensional data representations is the central issue in modern machine learning. In many models the data are represented by vectors in Euclidean spaces. The corresponding architectures build upon these representations by leveraging apparatus of linear algebra. However, it has been recently recognized that some ubiquitous types of data are more faithfully represented in negatively curved manifolds. We briefly overview recent research trends in this direction and explain the motivation. For further advances in hyperbolic deep learning it is necessary to enhance the corresponding mathematical framework. We introduce the notion of conformal barycenter in Poincaré balls. This point should be regarded as the mean (average) in hyperbolic balls. Furthermore, we derive the gradient descent algorithm for computing this point. For statistical modeling, we introduce the novel family of probability distributions in Poincaré balls and explain the sampling procedure and maximum likelihood estimation for this family. We emphasize significance of these mathematical results for future advances geometric deep learning.

Indecomposability of the median hypersimplex

Filip D. Jevtić
Mathematical Institute of the Serbian Academy of Sciences and Arts

Building on our previous work on Bier spheres we prove that the median hypersimplex $\Delta_{2k,k}$ is Minkowski indecomposable, i.e., that it cannot be expressed as a non-trivial Minkowski sum $\Delta_{2k,k} = P + Q$, where $P \neq \lambda\Delta_{2k,k} \neq Q$. We obtain as a corollary that $\Delta_{2k,k}$ represents a ray in the submodular cone, which is the deformation cone of the permutahedron. These results are obtained in ongoing joint work with R.T. Živaljević and M.Ž. Timotijević.

Markov chain model for spread and suppression of rumors

Jelena Jocković, Bojana Todić
Faculty of Mathematics, University of Belgrade

In the paper by Mahmoud (2020) the author proposes a discrete time Markov chain model for the spread of rumors in a fixed-size community, assuming that the rumor is manufactured by one person, and then spreads through one-to-one non-simultaneous interactions within the community. The person who hears the rumor will either believe it and spread it further (become the transmitter), or ignore it. This means that the rumor can only spread further, and eventually reach the entire community. We generalize the problem by introducing the possibility of rumor suppression, which is performed by the people in the community who are able to convince the person spreading the rumor that it is not true (and thus reduce the number of transmitters). We further generalize the problem by assigning arbitrary probabilities to each of the three types of responses to rumors, accounting for individual differences between communities. Our goal is to analyze the expected waiting time required for the rumor to reach a certain part of the population under this scenario by analyzing the properties of the fundamental matrix for the corresponding Markov chain.

On a sharp form of curvature conjecture for minimal graphs

David Kalaj
Faculty of Science and Mathematics, University of Montenegro

Recently, we resolved the longstanding Gaussian curvature problem ([2]) by establishing a sharp inequality $|\mathcal{K}| < c_0 = \frac{\pi^2}{2}$ for minimal graphs defined over the unit disk, evaluated at the point on the graph lying directly above the origin. The constant c_0 is called Heinz constant. Using this result, we derive an improved estimate for the Hopf constant c_1. Furthermore, we prove that for any given unit normal $\mathbf{n}$, there always exists a minimal graph over the unit disk, bending at coordinate directions, whose Gaussian curvature at the point above the origin is smaller—but arbitrarily close to—the Gaussian curvature of the corresponding Scherk-type surface associated with this normal, lying above a bicentric quadrilateral. This sharp inequality strengthens a classical result of Finn and Osserman, which applies to the specific case of the unit normal $(0, 0, 1)$.

[1] D. Kalaj. Curvature of minimal graphs. *Preprint, arXiv:2108.09447*, 2021.

[2] D. Kalaj and P. Melentijevic. Gaussian curvature conjecture for minimal graphs. *Duke Mathematical Journal*, 2025.

Náboj International Mathematical Competition

Mirjana Katić
Mathematical Grammar School, Belgrade

Náboj is an international mathematics competition in which Serbia participated for the first time last year. So far, a total of 13 countries have taken part. The competition is held separately in junior and senior divisions. It is a team competition, with each team consisting of five members. The duration of the competition is 120 minutes. The problems require comprehension, analytical thinking, and speed, as well as a certain level of intensity and resourcefulness.

Nonlinear regression with real algebraic varieties and their topology

Stephan Klaus
Oberwolfach Research Institute for Mathematics, Germany

The standard approach in topological data analysis is by persistent homology using the Čech or Vietoris-Rips complex for a point data cloud. This approach has a problem of high computational cost. In our talk we consider another possible method: approximation of the point cloud by nonlinear regression with real algebraic varieties. In addition, we give an overview on some results concerning the topology of real algebraic varieties.

Spectral radius subadditivity for integrals of operator-valued functions

Mihailo Krstić
Faculty of Mathematics, University of Belgrade

Let (Λ, Σ, μ) be a measurable space and let X be a Banach space. We denote by $r(A)$ the spectral radius of an operator $A \in \mathcal{B}(X)$. In [1] we prove that if a commuting family of operators $(T_\lambda)_{\lambda\in\Lambda}$ on X is Bochner integrable over Λ, then

$$r\left(\int_\Lambda^B T_\lambda \, d\mu(\lambda)\right) \leqslant \int_\Lambda r(T_\lambda) \, d\mu(\lambda).$$

This result extends the classical theorem on the subadditivity of the spectral radius for a finite set of commuting operators. In addition, we provide an example showing that Bochner integrability cannot be replaced by weaker forms of integrability considered in [2].

[1] H. Stanković. M. Krstić, *Spectral radius subadditivity for integrals of operator-valued functions*, accepted in Analysis Mathematica, (2025).

[2] M. Arsenović, M. Krstić. *A Normed Space of Weakly Integrable Operator-Valued Functions and Convergence Theorems*, Bulletin of the Iranian Mathematical Society, (2025), https://doi.org/10.1007/s41980-024-00965-x.

[3] J. Diestel, J. J. Uhl. *Vector Measures*, Amer. Math. Soc., Mat. Surveys and Monographs, Vol. **15**, (1977).

Several complex variables and CR geometry

Bernhard Lamel
Faculty of Mathematics, University of Vienna

We are giving a short introduction and a survey of recent results and open problems in CR geometry. The field has developed from questions in several complex variables first observed by Poincare, over questions surrounding the solvability of partial differential equations, in particular questions about the stability of Cauchy-Kowalevskaya solutions, and now is a rich area of research connecting geometry and analysis, and we aim to give a taste of the mathematics that has been developed in the area over the last decade.

Extended of Schatten p-norms and Nikiforov conjecture

Ivan Lazarević
Faculty of Civil Engineering, University of Belgrade

In this paper, we resolve the conjecture of V. Nikiforov, which states that if $p > 2$ and G is an undirected graph of order n with Schatten p-norm $\|G\|_p$, and K_n is the complete graph of order n, then $\|G\|_p^p \leq \|K_n\|_p^p$. We also provide a sharp upper bound for the sum of the p-th powers of the k largest singular values of a graph for $p \geq 2$, where the conjecture corresponds to the special case $k = n$. Furthermore, we establish a tight bound for the sum of the squares of the k largest singular values of a graph, including, in particular the case of the two largest eigenvalues. These results are then applied to prove Nikiforov's conjecture.

Bier spheres, toric manifolds and applications

Ivan Limonchenko
Mathematical Institute of the Serbian Academy of Sciences and Arts

A toric variety X of (complex) dimension n is a normal complex algebraic variety containing the algebraic torus $(\mathbb{C}^*)^n$ as a Zariski open subset in such a way that the natural action of $(\mathbb{C}^*)^n$ on itself extends to the action of $(\mathbb{C}^*)^n$ on X. A classical result of toric geometry is the fundamental correspondence between toric varieties X of complex dimension n and rational polyhedral fans Σ in $\mathbb{R}^n$, which allows us to write X_Σ for each toric variety under our consideration. Moreover, due to this correspondence, a toric variety X_Σ is a complete variety if and only if the fan Σ is complete, it is a nonsingular variety if and only if the fan Σ is regular, and it is a projective variety if and only if Σ is a normal fan of a lattice polytope. This provides a strong connection between combinatorics, topology and algebraic geometry.

In this talk, we are going to consider the class of PL spheres called Bier spheres and their combinatorial invariants (chromatic and Buchstaber numbers) from the viewpoint of toric geometry and topology. By definition, a Bier sphere $\mathrm{Bier}(K)$ of a simplicial complex K on $[m] := \{1, 2, \ldots, m\}$ with $m \geq 2$ different from the whole simplex $\Delta_{[m]}$ is an $(m-2)$-dimensional simplicial complex equal to the deleted join of K and its Alexander dual complex $K^\vee$. We will introduce the class of canonical toric manifolds X_K over simplicial complexes K, i.e. complete nonsingular toric varieties, arising from canonical complete regular fans Σ_K of Bier spheres $\mathrm{Bier}(K)$ based on the construction due to Jevtić, Timotijević and Živaljević. As direct applications of our construction, we will discuss a new topological proof of the Dehn-Sommerville relations for the face vector of a Bier sphere and a new geometrical approach to the Buchstaber-Panov problem on finding toric manifolds that are not quasitoric, which was solved by Suyama using resolution of singularities technics. Finally, we will introduce a criterion for a canonical real toric manifold over a simplicial complex to be orientable.

The talk is based on joint works with Marinko Timotijević (University of Kragujevac), Aleš Vavpetič (University of Ljubljana), and Rade Živaljević (Mathematical Institute SASA).

Plumbings and contact structures

Aleksandra Marinković
Faculty of Mathematics, University of Belgrade

A construction of 4-manifolds according to a plumbing graph was introduced in 60th's by Milnor and Wall, and later developed by Hirzebruch, Neumann etc. In this talk, I will recall this construction and, then, I will explain how to impose a symplectic structure with a contact type boundary on these manifolds. A contact structure on the boundary can be tight or overtwisted, depending on the self-intersection numbers of base surfaces. In the case of linear plumbings over spheres, the main tool to show tightness or overtwistedness of the contact boundary is Symplectic and Contact toric Geometry. I will also discuss further generalisations.

This talk is based on a joint work with Jo Nelson, Ana Rechtman, Laura Starkston, Shira Tanny and Lyua Wang.

The distance function and Lipschitz classes of mappings between metric spaces

Marijan Marković
Faculty of Science and Mathematics, University of Montenegro

We investigate when the local Lipschitz property of the function $g(z) = d_Y(f(z), A)$ implies the global Lipschitz property of the mapping $f : X \to Y$ between the metric spaces (X, d_X) and (Y, d_Y), where we have denoted by $d_Y(y, A)$ the distance of $y \in Y$ from the non-empty set $A \subseteq Y$. One of our consequences is that an analytic function on a uniform domain in a normed space belongs to the Lipschitz class if and only if its modulus satisfies the same condition; in the case of the unit disk this result is proved by K. Dyakonov. We use the recently established version of a classical theorem by Hardy and Littlewood for mappings between metric spaces. This talk contains result from the recent articles by the author [Marković, The Journal of Geometric Analysis **34** (2024)] and [Marković, Mathematika **71** (2025)].

On synergy of federated machine learning and blockchain against backdoor attacks

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Federated Learning (FL), is a distributed machine learning (ML) framework that enables participants to collaboratively train AI models while safeguarding data privacy by avoiding direct raw data exchange. Blockchain ensures secure, transparent, and tamper-proof data management by validating and recording transactions via consensus across network nodes. Main goals of the backdoor attacks are: (i) to make the model diverge from the main task; or (ii) to inject some trigger patterns into the global model to miss-classify the targeted class. The attack method (i) could be detected with the abnormal loss of global or local models on a validation set, since the model will not converge to the expected result. The attack method (ii) is more difficult to detect because the global model will not diverge from the main task.

This talk addresses:
- the main issues of FL and backdoor attack; and
- an approach based on dedicated employment of blockchain to support security of FL.

Finite Kurepa trees and left factorial hypothesis

Žarko Mijajlović
Faculty of Mathematics, University of Belgrade

This talk delves into two fascinating areas stemming from the work of Djuro Kurepa: the combinatorial properties of finite Kurepa trees and the intriguing Left Factorial Hypothesis. While seemingly distinct, both topics are deeply connected and highlight Kurepa's profound contributions to set theory, combinatorics, and number theory.

The first part of the talk will briefly address Kurepa's celebrated and *Left Factorial Hypothesis*. The left factorial $!n$ is defined as the sum $\sum_{k=0}^{n-1} k!$. Kurepa conjectured in 1971 that for any integer $n > 1$, the greatest common divisor of $!n$ and $n!$ is exactly 2. This hypothesis remains an open problem, captivating mathematicians for over half a century. We will provide a concise overview of the historical context and the significance of this hypothesis, drawing from the survey "Fifty years of Kurepa's $!n$ hypothesis" (Bulletin T. CLIV de l'Academie Serbe des sciences et des arts – 2021).

The primary focus of the second part will be on the combinatorial structure of *finite Kurepa trees*, drawing extensively from recent work on their partitions and automorphisms. A finite Kurepa tree, in this context, is understood as a finite rooted tree satisfying specific branching conditions that echo the properties of their infinite counterparts, particularly regarding the relationship between the number of vertices at each level and the total number of branches. We explore various ways to *partition* these trees, such as partitioning by branches, by levels, or by specific subtree structures. Understanding these partitions provides deep insights into the internal organization and complexity of these combinatorial objects. In particular, finding specific partitions would resolve !n hypothesis. Furthermore, we investigate the *automorphism groups* of finite Kurepa trees. The study of automorphisms reveals the symmetries inherent in these structures, shedding light on their unique combinatorial properties. We will discuss how the structure of these groups relates to the tree's branching patterns and how different partitioning schemes can influence the nature of its symmetries. Key results from our recent paper, "Partitions and Automorphisms of Finite Kurepa Trees" (accepted for Springer publication, preprint available ar Researchgate), will be presented, highlighting novel combinatorial techniques and characterizations of these finite structures. This part of the talk will emphasize the intricate interplay between the local branching behavior and global structural properties of finite Kurepa trees.

This talk aims to provide a comprehensive overview of these two interesting areas, offering combinatorial insights into finite Kurepa trees and in that context, a look at $!n$ hypothesis. Both versions stands as another testament to Kurepa's

ability to introduce new notions and pose fundamental and challenging problems across different mathematical disciplines.

Testing multivariate logisticity via weighted characteristic-function distances

Andjela Mijanovic Glusica
Faculty of Science and Mathematics, University of Montenegro

We present an affine-invariant goodness-of-fit test for multivariate logistic distributions based on a weighted L^2 distance between the empirical characteristic function (ECF) of standardized data and the theoretical characteristic function of the standard multivariate logistic model. The statistic employs a radial weight $e^{-a||t||}$ and admits a closed-form representation via radial (Hankel) transforms, yielding computable expressions in terms of special functions. We derive the limiting distribution under the null and under contiguous alternatives, establish affine invariance, and provide Monte Carlo critical values. Extensive simulations show accurate size and strong power relative to multivariate Kolmogorov–Smirnov, energy, and χ^2 tests, especially in higher dimensions and under heavy-tailed alternatives. Applications to real datasets illustrate practical robustness and ease of use. Open-source Matlab and R code facilitates replication and adoption.

Determining probabilistic properties of generalized stochastic variables via moment methods and chaos expansions

Danijela Milenković
Faculty of Pharmacy, University of Belgrade

Dora Seleši
Faculty of Sciences, University of Novi Sad

The polynomial chaos expansion method has become a powerful tool for solving stochastic differential equations in recent years. It represents solutions using orthogonal series, raising two key challenges: how to compute fundamental probabilistic properties (such as expectation, variance, finite-dimensional distributions) and how to model input parameters (e.g., coefficients, driving forces, initial conditions) within this framework. This work addresses both. Using the Wiener-Itô chaos expansion with Hermite polynomials, we derive formulas to compute probability distributions directly from expansion coefficients. Since moments and moment determinacy play a critical role in these expressions, we also provide a formula for evaluating nth order moments from the chaos expansion coefficients. Additionally, we present an algorithm for constructing the Wiener-Itô chaos expansion of known probability distributions using generalized Legendre polynomials.

An isomorphism between classical and quantum mechanics by means of the Heisenberg algebra

Miloš Milovanović
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Nicoletta Saulig
Faculty of Engineering of the Juraj Dobrila, University of Pula

The fact that Galilean transformations form the Heisenberg group is used to constitute an isomorphism between classical and quantum mechanics. In that manner, the quantum uncertainty between position and velocity is reflected by a classical one between time and velocity. Other concepts should be consistently transformed as well in order to implement quantum mechanics onto the classical instance. The position is also replaced by the energy operator which makes quantum dynamics governed by the Schrödinger equation to be embedded into classical kinematics. Classical dynamics has required a transition toward statistical quantum mechanics which permits the definition of a time operator that is a concept having no classical counterpart. The formulation offers a resolution of the measurement problem which is a crucial challenge that is shared by both classical and quantum mechanics.

Weakly C-chain separated sets in a topological space

Zoran Misajleski
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Let $\mathcal{U}$ be a covering of a topological space X in X and $x, y \in X$. A **chain** in $\mathcal{U}$ that connects x and y is a finite sequence of elements of $\mathcal{U}$ such that x belongs to the first element of the sequence, y to the last, and the intersection of any two consecutive elements of the sequence is nonempty. Let A and B be nonempty subsets of the topological space X. The sets A and B are **weakly C-chain separated** in X, if for every point $x \in A$ and every $y \in B$, there exists a clopen covering $\mathcal{U} = \mathcal{U}_{xy}$ of X such that there is no chain in $\mathcal{U}$ that connects x and y. The sets A and B are **C-chain separated** in X, if there exists a clopen covering $\mathcal{U}$ of X such that for every point $x \in X$ and every $y \in Y$, there is no chain in $\mathcal{U}$ that connects x and y. In this presentation, we will study the properties of these sets.

Teaching classical constructions today: compass, ruler, and dynamic geometry for secondary students

Majda Mrdak
Slobodan Škerović Gymnasium, Podgorica

This talk reports on a teaching design from a master's thesis on compass-and-straightedge constructions and their algebraic counterparts, aimed at upper-secondary classrooms and contest clubs. Classical theorems—Mohr–Mascheroni (compass only) and Poncelet–Steiner (straightedge only with a given circle and its center)—are used to model rigorous reasoning and proof. Students move from paper constructions to dynamic software (GeoGebra/Cinderella). I argue that these tools are not "gadgets" but instruments for conjecturing, visual feedback, and systematic error detection in short lesson sequences.

A central focus is making explicit the difference between geometric constructibility (what can be built with compass and ruler) and algebraic constructibility (numbers obtained via successive quadratic field extensions). This distinction clarifies why some regular polygons (e.g., the 80-gon) are constructible and why the classical trio—angle trisection, cube duplication, and circle squaring—is impossible in the classical framework. The "power of a point" is a highlight: a straightedge-only construction of the intersection of two circles, with dynamic verification tasks aligned with Olympiad-style thinking.

The approach is particularly valuable for students in specialized mathematics tracks who possess the algebraic fluency to connect Euclidean steps with field-theoretic ideas. The session outlines learning objectives, sample prompts for formative assessment, and options for adapting activities for competition preparation—emphasizing pedagogical rationale rather than distributing fixed handouts.

Multi-criteria decision making based on the energy of fuzzy soft sets: applications in practice

Ljubica Mudrić-Staniškovski
Faculty of Engineering, University of Kragujevac

This paper presents a novel approach to multi-criteria decision making based on the theory of fuzzy soft sets. We introduce numerical characteristics — energy and λ-energy — derived from the singular values of the associated matrices, enabling systematic ranking of alternatives under uncertainty. The methodology is demonstrated through practical examples, including candidate selection and cloud platform evaluation, showing that the proposed framework provides consistent, transparent, and applicable results. The proposed approach offers a flexible and reliable tool for decision makers in various complex and uncertain environments.

Comparing online and traditional mathematics teaching – lessons from practice

Ljubica Mudrić-Staniškovski
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The COVID-19 pandemic imposed a sudden and comprehensive implementation of online teaching at all levels of education. Mathematics teaching, as a subject traditionally based on direct interaction between teachers and students, faced numerous challenges but also new opportunities. This paper presents the author's personal experiences in the realization of teaching in different educational contexts – from recording lessons for the national television RTS during school closures, through teaching in primary and secondary schools, to online teaching at the university level during faculty blockades in Serbia. The advantages and limitations of both teaching models are analyzed, with special emphasis on the possibilities for motivation, individualization, and monitoring the progress of pupils and students. The aim of the paper is to show how experiences gained in crisis conditions can serve as a basis for improving the quality of mathematics teaching in regular circumstances, by combining traditional and online approaches in the future.

Surgery for Seifert and graph manifolds

Jovana Nikolić, Vuk Ovaskainen
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Zoran Petrić
Mathematical Institute of the Serbian Academy of Sciences and Arts

A Seifert manifold is a 3-manifold represented as a union of pairwise disjoint circles, which must fit nicely together. A surgery consists of extracting a solid torus from a 3-manifold and sewing it back in a different manner. The aim of this talk is to present Seifert manifolds as surgery data in S^3 in a way that not only the topological structure, but the whole Seifert fibered structure is visible. This requires to restrict the Kirby calculus so that the accepted moves do not change the latter structure. The ultimate goal is to generalise this to graph manifolds (manifolds that can be split by a set of disjoint tori into Seifert manifolds).

Overview of anti-Gaussian quadrature formulas on the space of multiple orthogonal polynomials with nearly diagonal multi-indeces

Nevena Z. Petrović
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Multiple orthogonal polynomials represent a generalization of orthogonal polynomials in the sense that they satisfy the orthogonality conditions with respect to $r \in \mathbb{N}$ different weight functions simultaneously. In this presentation, we will provide an overview of the set of anti-Gaussian quadrature formulas for the optimal set of quadrature formulas in Borges' sense on the spaces of algebraic and trigonometric polynomials, while limiting ourselves to nearly diagonal multi-indices. The corresponding multiple orthogonal polynomials that arise in the construction of these quadrature formulas satisfy the orthogonality conditions with respect to r bilinear forms that naturally emerge from the mentioned constructions.

Multivariate distributions with proportional hazard rate marginal and its dependence modelling

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We introduce a new family of multivariate distributions, so called the multi- variate proportional hazard rate family of distributions and study its properties. We derive multivariate dependence properties based on a survival function and based on a survival copula function. Multivariate dependence properties based on a survival function are given by multivariate total positivity of order 2, right corner set increasing, smaller in lower orthant order and right tail increasing. Multivariate dependence properties based on a survival copula function are some coefficients of concordance. The survival copula function is used to model this family of multivariate distributions.

On the duality principle in Osserman manifolds

Zoran Rakić
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Let (M, g) be a pseudo-Riemannian manifold, with curvature tensor R. The Jacobi operator R_X is the symmetric endomorphism of T_pM defined by $R_X(Y) = R(Y, X)X$. In Riemannian settings, if M is locally a rank-one symmetric space or if M is flat, then the local isometry group acts transitively on the unit sphere bundle SM and hence the eigenvalues of R_X are constant on the unit sphere bundle SM. Osserman in the late eighties, wondered if the converse held; this question is usually known as the *Osserman conjecture*.

In the first part of the lecture we will give an overview of Osserman type problems in the pseudo-Riemannian geometry. The second part is devoted to the equivalence of the Osserman pointwise condition and the duality principle. This part of the lecture consists of results, which are obtained in collaboration with Yury Nikolayevsky and Vladica Andrejić.

Non-additive measures and integrals and applications in face recognition

Nebojša M. Ralević
Faculty of Technical Sciences, University of Novi Sad

Nataša Milosavljević
Faculty of Agriculture, University of Belgrade

Ljubo Nedović
Faculty of Technical Sciences, University of Novi Sad

This paper presents the concepts of different non-additive measures, especially some classes of c-credibility measures, as well as their properties. Integrals based on such measures actually represent a very important class of aggregation functions that play a very important role in applications. Properties are shown for one such class of integrals.

Recognizing a human face in a digital image is a very important application of image processing and the mathematical knowledge used in it, for instance, the theory of non-additive integrals. Thus, the aggregation of the results of the comparison of the similarity of each part of the face individually, gives a coefficient overlapping of two observed faces. The effectiveness of this method is compared with similar methods on selected image databases.

The conditional p-next center problem

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The p-center problem (PCP) is one of the most studied combinatorial optimization problems in location theory, as it is widely used in the planning of emergency service networks, supply chains, telecommunication networks, etc. Given a set of n locations and distances between them, the objective of the PCP is to select p locations for opening the service centers and assign each of the remaining $n - p$ locations (users) to the nearest center, such that the maximal distance of a user to its assigned center is minimized.

Up to now, various modifications and extensions of the PCP have been developed to capture different real-life situations. From a practical point of view, one of the most important modifications of the PCP is the p-next center problem (PNCP), which takes into account the failure of one or more centers due to an accident or other problem. If a center suddenly fails, its users are redirected to the center closest to the failed center. The goal of the PNCP is to minimize the maximal distance that a user must travel to be served.

The conditional p-next center problem (CPNCP) is a modification of the PNCP that deals with the expansion of a service network. In addition to the possibility that some centers may fail, the CPNCP assumes that q existing centers are kept and p new centers must be opened. The considered CPNCP is formulated as an Integer Linear Program that is further used within the framework of exact solver CPLEX to find optimal or feasible solutions on modified test problems from the literature. In order to analyze the effects of using PNCP and CPNCP as two different strategies in service network planning, the CPNCP objective function values for the given p and q are compared with the corresponding the PNCP objective function values when $p+q$ centers need to be opened. The obtained results are analyzed to determine the potential losses if the existing facilities are kept when expanding a service network.

A novel first-order INAR model in random environment

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Although the majority of existing INAR models are stationary, it is well recognized that numerous natural processes cannot be adequately described using stationary frameworks. Initial efforts to address this limitation led to the development of the INAR model in a random environment. We present a new first-order integer-valued autoregressive model set in a random environment. The proposed model captures dynamic shifts driven by an underlying control mechanism, allowing flexible transitions between autoregressive and memoryless regimes. Through this mechanism, each element of the model is assigned one of two possible behaviors: it can either display the features of an INAR(1) process or behave as part of a sequence of independent and identically distributed random variables. The model is

constructed using a modified negative binomial operator. The most important properties of the introduced model are derived. The quality of the estimates is evaluated using simulated values. Finally, the possible application of the introduced model to a real dataset is provided. The results show that the model provides an improved fit compared to conventional INAR-type models in the presence of structural shifts and nonstationarity.

Complex Grassmann manifolds $G_{n,2}$ and compactifications of the moduli space $\mathcal{M}_{0,n}$

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For the canonical action of the compact torus T^n on a complex Grassmann manifold $G_{n,2}$ we introduced the universal space of parameters $\mathcal{F}_n$, which is a smooth compact manifold. This action produces as well a family $\{F_\omega\}$ of smooth, compact manifolds in the orbit space $G_{n,2}/T^n$. The manifold $\mathcal{F}_n$ and the manifolds $\{F_\omega\}$ play crucial role in the description of this orbit space. We explain how these manifolds are related to the compactifications of the moduli space $\mathcal{M}_{0,n}$ of the n marked distinct points in the complex projective line $\mathbb{C}P^1$, which are given by the moduli spaces of weighted pointed stable curves. We show that the natural continuous projections $\mathcal{F}_n \to F_\omega$, which are as well key ingredients in the description of $G_{n,2}/T^n$, realize the reduction morphisms between the corresponding moduli spaces. In this way it singles out the category consisting of moduli spaces of weighted pointed stable curves which can be realized by the topological category arising from T^n-action on $G_{n,2}$. In this context we discuss two distinguished moduli compactifications of $\mathcal{M}_{0,n}$, that is the Deligne-Mumford $\overline{\mathcal{M}}_{0,n}$ compactification and the Losev-Manin $\bar{L}_{0,n}$ compactification.

The talk is based on joint papers with Victor M. Buchstaber.

Markov chain model for the spread of rumors with group transmission

Bojana Todić, Jelena Jocković
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In the paper by Mahmoud (2020) the author proposes a discrete time Markov chain model for the spread of rumors in a fixed-size community, assuming that the rumor is generated by one person, and then spreads through one-to-one non-simultaneous interactions within the community. The latter assumption is unrealistic in many real-world situations (such as, for example, office gossip, where an informed person may spread the news to all of their office mates at once). We generalize the scenario analyzed by Mahmoud by introducing the possibility of group transmission, which means that an informed person can pass the rumor to multiple people (group of recipients) within a single interaction. Next, we further generalize

the problem by assigning different distributions to the size of the group of recipients, and calculate the expected waiting time required for the rumor to reach a certain part of the population using the fundamental matrix for the corresponding Markov chain. Finally, we discuss the direct connection of the proposed scenario to modeling the spread of infectious diseases.

On the approximation of functions within metric frameworks

Daniel Velinov
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In this talk, the metrical approximations of functions $F : \Lambda \times X \to X$ by trigonometric polynomials and ρ-periodic type functions, where $\emptyset \neq \Lambda \subseteq \mathbb{R}^n$, X and Y are complex Banach spaces, and ρ is a general binary relation on Y are analyzed. We establish a range of structural properties of the newly introduced function spaces and explore their approximation behavior. Furthermore, we demonstrate the applicability of our theoretical framework to various classes of equations, including abstract Volterra integro-differential equations and partial differential equations.

Assessing error distributions in circular regression models

Mirjana Veljović
Faculty of Mathematics, University of Belgrade

We study goodness-of-fit testing for error distributions in regression models with circular responses. The main goal is to test whether the model errors follow the von Mises distribution, one of the most prominent circular distributions. To this end, we adapt a recently introduced goodness-of-fit test for circular i.i.d. data to the regression setting and derive its asymptotic properties. The finite-sample behavior of the test is further investigated through extensive simulations, where we also compare its performance with existing methods. Finally, the practical relevance of the procedure is illustrated through applications to several real datasets assessing model fit.

Inverse spectral problems; From Ambarzumian's theorem to Theorem of uniqueness

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Inverse spectral theory its roots back to classical results such as Borg's two spectra theorem and Ambarzumian's theorem, which provide foundational insights into how the spectrum of a Sturm-Liouville operators determines its potential.
In his famous paper "Uber eine Frage der Eigenwerttheeorie" from (1929) Ambarzumian describes the exceptional case in wich the spectrum of boundary value problem generated with Sturm-Liouville operator under Neumann boundary conditions uniquely determines potential.
In general, the specification of the spectrum does not uniquely determined potential. Borg(1946) proved Theorem of uniqueness from two spectra of the boundary value problem defined with Sturm-Liouville operator under Robin boundary conditions.
In addition to this result, many mathematicians contributed to the determination potential functions or proved the uniqueness of the potential based on other spectral characteristics like spectral function, spectral data or Weyl function and we can say that inverse problem for Sturm-Liouville differential operators is fully solved.
One of the most important direction for further research was BVPs generated with a Sturm-Liouville type differential equation with a constant delay and one of the first resulat was given by Freiling and Yurko (2012) and they proved Ambarzumian type theorem for differential operators defined with Sturm-Liouville equation with constant delay under Dirichlet-Neumann boundary conditions from two spectra. Compared with Ambarzumian's theorem it could be expected that Borg-type inverse problem for this BVP will be more complicated than in the case of classical Sturm-Liouville operator. In several papers authors solve Borg-type inverse spectral problems for this class of the boundary value problems and they give complete answer for which value of the delay Theorem of uniqueness from two spectra is true or not true.
There has recently been increasing interest in boundary value problems generated by Sturm–Liouville operators with more than one constant delay. In the case with two constant delays an attempt was made to answer the question of how many spectra are needed for Ambarzumian's theorem to be correct, but until now we don't have answer. Also, we don't have complete answer when potential function is unique ordered.
The focus of this research is on the boundary value problems generated with differential equation Sturm-Liouville-type two constant delay. The Ambarzumian type theorem for operator with two constant delays from four spectra will be proven and a partial answer to the question of when the Theorem of uniqueness is true and when it is not will be given.

Optimal pointwise estimate for the derivative of $\alpha-$harmonic functions

Djordjije Vujadinović
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For $\alpha > -1$ we established the optimal estimate of the type $|Df(z)| \leq \phi(|r|)\|f^*\|_p$ where f is $\alpha-$harmonic mapping defined in the unit disc belonging to the Hardy space h^p, $p \geq 1$, $\alpha + \frac{2}{p} \geq 0$ and f^* is its boundary function.

Gödel on epistemology

Slobodan Vujošević
Mathematical Institute of the Serbian Academy of Sciences and Arts

Gödel's view on the objectivity of the conceptual world originally referred only to mathematical concepts, but during the 1940s it was extended to all concepts. For the needs of mathematical realism, as a philosophy of mathematics, he formulated an appropriate mathematical epistemology. But since "logic is primarily a science of concepts and only partially concerns mathematics and its entities," for Gödel the key philosophical problem was establishing an epistemology of the conceptual world. In this lecture we present Gödel's mathematical epistemology as well as certain controversies regarding his standpoints about the general epistemology of concepts.

Application of optimization to binary logistic regression

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This paper explores the application of optimization techniques in machine learning, with a focus on binary logistic regression. We provide an illustrative implementation of two widely used optimization methods: Gradient Descent and Newton's Method. The methods are applied in the context of a Mathematics 3 exam at the Faculty of Electrical Engineering (ETF), serving as a practical example of how these techniques operate in real-world academic problems. Additionally, we compare the performance, convergence speed, and computational efficiency of the two methods. The impact of regularization and its degree on model performance is also analyzed, highlighting its role in preventing overfitting and improving generalization. This study aims to demonstrate the practical importance of optimization in training classification models and the trade-offs involved in method selection and regularization strength.

A polyhedral miscellany

Rade T. Živaljević
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Our objective in "Polyhedral Miscellany" is to exhibit and study examples, illustrating special (and often non-standard) role of polytopes and polyhedra in different areas of mathematics.

The first theme is a report about a (recently discovered) connection of *Icosahedron Avatars*, as presented in the animation `https://vimeo.com/user3279879`, with *Kepler-Poinsot star-polyhedra*. This animation was originally created (circa 2012) by Dušan Živaljević (with a mathematical assistance of the author) for the project of popularization of mathematics, supported by the Serbian Ministry of Science and Technology and Mathematical Institute SASA (Belgrade).

One of the goals of *welfare economics* is to divide a resource among players in an *envy-free* manner. Envy-freeness is the principle where every player feels that their share is at least as good as the share of any other player, and thus nobody feels envy. Here we meet *Voronoi polytopes* which allow us to obtain sharp lower bounds for the number of envy-free division in the classical Woodall-Stormquist setting, i.e. for players with standard KKM preferences.

Simple games (von Neumann and Morgenstern 1944; Shapley 1962; Taylor and Zwicker 1999, etc.) are mathematical objects originally used in mathematical economics (cooperative game theory) to model the distribution of power among coalitions of players. Each simplicial complex $K \subsetneq 2^{[n]}$ is associated an $(n-2)$-dimensional, combinatorial sphere on (at most) $2n$-vertices, called *Bier sphere* $Bier(K)$ (named after T. Bier). Here we discuss an analogy between (non)weighted simple games and (non)polytopal Bier spheres (canonical fans).


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