Fixed Point Theory and Applications
a SpringerOpen Journal

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Some Suzuki-type fixed point theorems for generalized multivalued mappings and applications

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Abstract

In this article we obtain a Suzuki-type generalization of a fixed point theorem for generalized multivalued mappings of Ćirić (Matematički Vesnik, **9**(24), 265-272, 1972). The obtained results extend furthermore the recently developed Kikkawa-Suzuki-type contractions. Applications to certain functional equations arising in dynamic programming are also considered.

Keywords: Complete metric space, fixed point, multivalued mapping, functional equation

1 Introduction and preliminaries

In 2008 Suzuki [1] introduced a new type of mappings which generalize the well-known Banach contraction principle [2]. Some others [3] generalized Kannan mappings [4].

Theorem 1.1. *(Kikkawa and Suzuki* [3]*) Let T be a mapping on complete metric space (X, d) and let ϕ be a non-increasing function from $[0, 1)$ into $(1/2, 1]$ defined by*

$$\varphi(r) = \begin{cases} 1, & \text{if } 0 \le r \le \dfrac{1}{\sqrt{2}}, \\ \dfrac{1}{1+r}, & \text{if } \dfrac{1}{\sqrt{2}} \le r < 1. \end{cases}$$

Let $\alpha \in [0, 1/2)$ and $r = \alpha/(1 - \alpha) \in [0, 1)$. Suppose that

$$\varphi(r)d(x, Tx) \le d(x, y) \text{ implies } d(Tx, Ty) \le \alpha d(x, Tx) + \alpha d(y, Ty) \tag{1}$$

for all $x, y \in X$. Then, T has a unique fixed point z, and $\lim_n T^n x = z$ holds for every $x \in X$.

Theorem 1.2. *(Kikkawa and Suzuki* [3]*) Let T be a mapping on complete metric space (X, d) and θ be a nonincreasing function from $[0, 1)$ onto $(1/2, 1]$ defined by*

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \le r \le \frac{1}{2}(\sqrt{5}-1), \\ \dfrac{1-r}{r^2} & \text{if } \frac{1}{2}(\sqrt{5}-1) \le r \le \dfrac{1}{\sqrt{2}}, \\ \dfrac{1}{1+r} & \text{if } \dfrac{1}{\sqrt{2}} \le r < 1. \end{cases}$$

Suppose that there exists $r \in [0, 1)$ *such that*

$$\theta(r)d(x, Tx) \leq d(x, y) \text{ implies } d(Tx, Ty) \leq r \max\{d(x, Tx), d(y, Ty)\} \tag{2}$$

for all $x, y \in X$. *Then,* T *has a unique fixed point* z, *and* $\lim_n T^n x = z$ *holds for every* $x \in X$.

On the other hand, Nadler [5] proved multivalued extension of the Banach contraction theorem.

Theorem 1.3. *(Nadler* [5]*) Let* (X, d) *be a complete metric space and let* T *be a mapping from* X *into* $CB(X)$. *Assume that there exists* $r \in [0, 1)$ *such that*

$$H(Tx, Ty) \leq rd(x, y)$$

for all $x, y \in X$. *Then, there exists* $z \in X$ *such that* $z \in Tz$.

Many fixed point theorems have been proved by various authors as generalizations of the Nadler's theorem (see [6-9]). One of the general fixed point theorems for a generalized multivalued mappings appears in [10].

The following result is a generalization of Nadler [5].

Theorem 1.4. *(Kikkawa and Suzuki* [11]*) Let* (X, d) *be a complete metric space, and let* T *be a mapping from* X *into* $CB(X)$. *Define a strictly decreasing function* η *from* $[0, 1)$ *onto* $(1/2, 1]$ *by*

$$\eta(r) = \frac{1}{1+r}$$

and assume that there exists $r \in [0, 1)$ *such that*

$$\eta(r)d(x, Tx) \leq d(x, y) \text{ implies } H(Tx, Ty) \leq rd(x, y)$$

for all $x, y \in X$. *Then, there exists* $z \in X$ *such that* $z \in Tz$.

In this article we obtain a Kikkawa-Suzuki-type fixed point theorem for generalized multivalued mappings considered in [10]. The result obtained here complement and extend some previous theorems about multivalued contractions. In addition, using our result, we proved the existence and uniqueness of solutions for certain class of functional equations arising in dynamic programming.

2 Main results

Let (X, d) be a metric space. We denote by $CB(X)$ the family of all nonempty, closed, bounded subsets of X. Let $H(\cdot, \cdot)$ be the Hausdorff metric, that is,

$$H(A, B) = \max\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(A, b)\}$$

for $A, B \in CB(X)$, where $d(x, B) = \inf_{y \in B} d(x, y)$.

Now, we will prove our main result.

Theorem 2.1. *Define a nonincreasing function* ϕ *from* $[0, 1)$ *into* $(0, 1]$ *by*

$$\varphi(r) = \begin{cases} 1, & \text{if } 0 \leq r < \frac{1}{2}, \\ 1 - r, & \text{if } \frac{1}{2} \leq r < 1. \end{cases}$$

Let (X, d) *be a complete metric space and* T *be a mapping from* X *into* $CB(X)$. *Assume that there exists* $r \in [0, 1)$ *such that* $\phi(r)d(x, Tx) \leq d(x, y)$ *implies*

$$H(Tx, Ty) \leq r \cdot \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\} \tag{3}$$

for all $x, y \in X$*. Then, there exists* $z \in X$ *such that* $z \in Tz$.

Proof.

1. Let r_1 be such a real number that $0 \leq r < r_1 < 1$, and $u_1 \in X$ and $u_2 \in Tu_1$ be arbitrary. Since $u_2 \in Tu_1$, then $d(u_2, Tu_2) \leq H(Tu_1, Tu_2)$ and, as $\phi(r) < 1$,

$$\varphi(r)d(u_1, Tu_1) \leq d(u_1, Tu_1) \leq d(u_1, u_2).$$

Thus, from the assumption (3),we have

$$\begin{aligned} d(u_2, Tu_2) &\leq H(Tu_1, Tu_2) \\ &\leq r \cdot \max\left\{d(u_1,u_2), d(u_1,Tu_1), d(u_2,Tu_2), \frac{d(u_1,Tu_2)+0}{2}\right\} \\ &\leq r \cdot \max\left\{d(u_1,u_2), d(u_2,Tu_2), \frac{d(u_1,u_2)+d(u_2,Tu_2)}{2}\right\}. \end{aligned}$$

Hence, as $r < 1$, we have $d(u_2, Tu_2) \leq rd(u_1, u_2)$. Hence, there exists $u_3 \in Tu_2$ such that $d(u_2, u_3) \leq r_1 d(u_1, u_2)$. Thus, we can construct such a sequence $\{u_n\}$ in X that

$$u_{n+1} \in Tu_n \text{ and } d(u_{n+1}, u_{n+2}) \leq r_1 d(u_n, u_{n+1}).$$

Then, we have

$$\sum_{n=1}^{\infty} d(u_n, u_{n+1}) \leq \sum_{n=1}^{\infty} r_1^{n-1} d(u_1, u_2) < \infty.$$

Hence, we conclude that $\{u_n\}$ is a Cauchy sequence. Since X is complete, there is some point $z \in X$ such that

$$\lim_{n\to\infty} u_n = z.$$

2. Now, we will show that

$$d(z, Tx) \leq r \cdot \max\{d(z,x), d(x,Tx)\} \text{ for all } x \in X\backslash\{z\}. \tag{4}$$

Since $u_n \to z$, there exists $n_0 \in N$ such that $d(z, u_n) \leq (1/3)d(z, x)$ for all $n \geq n_0$. Then, we have

$$\begin{aligned} \varphi(r)\, d(u_n, Tu_n) &\leq d(u_n, Tu_n) \\ &\leq d(u_n, u_{n+1}) \\ &\leq d(u_n, z) + d(u_{n+1}, z) \\ &\leq \frac{2}{3} d(x, z). \end{aligned}$$

Thus,

$$\varphi(r)\, d(u_n, Tu_n) \leq \frac{2}{3} d(x, z). \tag{5}$$

Since

$$\begin{aligned}\frac{2}{3}d(x,z) &= d(x,z) - \frac{1}{3}d(x,z)\\ &\le d(x,z) - d(u_n,z)\\ &\le d(u_n,x),\end{aligned}$$

from (5), we have $\phi(r)\, d(u_n, Tu_n) \le d(u_n, x)$. Then, from (3),

$$\begin{aligned}H(Tu_n,Tx) \le\ & r\cdot\max\left\{d(u_n,x), d(u_n,Tu_n), d(x,Tx),\right.\\ &\left.\frac{d(u_n,Tx)+d(x,Tu_n)}{2}\right\}.\end{aligned} \tag{6}$$

Since $u_{n+1} \in Tu_n$, then

$$d(u_{n+1},Tx) \le H(Tu_n,Tx) \text{ and } d(u_n,Tu_n) \le d(u_n,u_{n+1}).$$

Hence, from (6), we get

$$d(u_{n+1},Tx) \le r\cdot\max\left\{d(u_n,x), d(u_n,u_{n+1}), d(x,Tx), \frac{d(u_n,Tx)+d(x,u_{n+1})}{2}\right\}$$

for all $n \in N$ with $n \ge n_0$. Letting n tend to ∞, we obtain (4).

3. Now, we will show that $z \in Tz$.

3.1. First, we consider the case $0 \le r < \frac{1}{2}$. Suppose, on the contrary, that $z \notin Tz$. Let $a \in Tz$ be such that $2rd(a, z) < d(z, Tz)$. Since $a \in Tz$ implies $a \ne z$, then from (4) we have

$$d(z,Ta) \le r\max\{d(z,a), d(a,Ta)\}.$$

On the other hand, since $\phi(r)\, d(z, Tz) \le d(z, Tz) \le d(z, a)$, then from (3) we have

$$\begin{aligned}H(Tz,Ta) &\le r\cdot\max\left\{d(z,a), d(z,Tz), d(a,Ta), \frac{d(z,Ta)+0}{2}\right\}\\ &\le r\max\left\{d(z,a), d(z,Tz), d(a,Ta)\right\}\\ &\le r\max\left\{d(z,a), d(a,Ta)\right\}.\end{aligned}$$

Hence,

$$d(a,Ta) \le H(Tz,Ta) \le r\max\left\{d(z,a), d(a,Ta)\right\}.$$

Hence, $d(a, Ta) \le rd(z, a) < d(z, a)$, and from (7), we have $d(z, Ta) \le rd(z, a)$. Therefore, we obtain

$$\begin{aligned}d(z,Tz) &\le d(z,Ta) + H(Ta,Tz)\\ &\le d(z,Ta) + r\max\left\{d(z,a), d(a,Ta)\right\}\\ &\le 2rd(z,a)\\ &< d(z,Tz).\end{aligned}$$

This is a contradiction. As a result, we have $z \in Tz$.

3.2. Now, we consider the case $\frac{1}{2} \le r < 1$. We will first prove

$$H(Tx,Tz) \le r\max\left\{d(x,z), d(x,Tx), d(z,Tz), \frac{d(,Tx)+d(z,Tx)}{2}\right\} \tag{8}$$

for all $x \in X$. If $x = z$, then the previous obviously holds. Hence, let us assume $x \neq z$. Then, for every $n \in N$, there exists a sequence $y_n \in Tx$ such that $d(z, y_n) \leq d(z, Tx) + (1/n)d(x, z)$. Using (4), we have for all $n \in N$

$$\begin{aligned} d(x, Tx) \leq\ & d(x, y_n) \\ \leq\ & d(x, z) + d(z, y_n) \\ \leq\ & d(x, z) + d(z, Tx) + \frac{1}{n} d(x, z) \\ \leq\ & d(x, z) + r \max\{d(x, z), d(x, Tx)\} + \frac{1}{n} d(x, z). \end{aligned}$$

If $d(x, z) \geq d(x, Tx)$, then

$$d(x, Tx) \leq d(x, z) + rd(x, z) + \frac{1}{n} d(x, z) = \left(1 + r + \frac{1}{n}\right) d(x, z).$$

Letting n tend to ∞, we have $d(x, Tx) \leq (r + 1)d(x, z)$. Thus,

$$\varphi(r)d(x, Tx) = (1 - r)d(x, Tx) \leq \frac{1}{r + 1} d(x, Tx) \leq d(x, z)$$

and from (3), we have (8).

If $d(x, z) < d(x, Tx)$, then

$$d(x, Tx) \leq d(x, z) + rd(x, Tx) + \frac{1}{n} d(x, z)$$

and therefore,

$$(1 - r)d(x, Tx) \leq \left(1 + \frac{1}{n}\right) d(x, z).$$

Letting n tend to ∞, we have $\phi(r)d(x, T) \leq d(x, z)$ and thus, from (3), we again have (8).

Finally, from (8), we obtain

$$\begin{aligned} d(z, Tz) =\ & \lim_{n \to \infty} d(u_{n+1}, Tz) \\ \leq\ & \lim_{n \to \infty} r \max\left\{d(u_n, z), d(u_n, Tu_n), d(z, Tz), \frac{d(u_n, Tz) + d(z, Tu_n)}{2}\right\} \\ \leq\ & \lim_{n \to \infty} r \max\left\{d(u_n, z), d(u_n, u_{n+1}), d(z, Tz), \frac{d(u_n, Tz) + d(z, u_{n+1})}{2}\right\} \\ =\ & rd(z, Tz). \end{aligned}$$

Hence, as $r < 1$, we obtain $d(z, Tz) = 0$. Since Tz is closed, $z \in Tz$.

Hence, we have shown that $z \in Tz$ in all cases, which completes the proof. □

Remark. The Theorem 2.1 provides the answer to the Question 1 posed in [12].

Corollary 2.1. *Let* (X, d) *be a complete metric space and* T *be a mapping from* X *into* $CB(X)$.

Assume that there exists $r \in [0, 1)$ *such that* $\phi(r)d(x, Tx) \leq d(x, y)$ *implies*

$$H(Tx, Ty) \leq r \max\{d(x, y), d(x, Tx), d(y, Ty)\} \tag{9}$$

for all $x, y \in X$, *where the function* ϕ *is defined as in Theorem 2.1. Then, there exists* $z \in X$ *such that* $z \in Tz$.

Proof. It comes from Theorem 2.1 since (9) implies (3). □

The Corollary 2.1 is the multivalued mapping generalization of the Theorem 2.2 of Kikkawa and Suzuki [3], and therefore of the Kannan fixed point theorem [4] for generalized Kannan mappings. Also, it is the generalization of the Theorem 2.1 of Damjanović and Đorić [13].

From the Corollary 2.1, we obtain an another corollary:

Corollary 2.2. *Let* (X, d) *be a complete metric space and* T *be a mapping from* X *into* $CB(X)$.

Let $\alpha \in [0, 1/3)$ *and* $r = 3\alpha$. *Suppose that there exists* $r \in [0, 1)$ *such that*

$$\varphi(r)d(x, Tx) \leq d(x, y) \quad \textit{implies} \quad H(Tx, Ty) \leq \alpha d(x, y) + \alpha d(x, Tx) + \alpha d(y, Ty)$$

for all $x, y \in X$, *where the function* ϕ *is defined as in Theorem 2.1. Then, there exists* $z \in X$ *such that* $z \in Tz$.

Considering T as a single-valued mapping, we have the following result:

Corollary 2.3. *Let* (X, d) *be a complete metric space and* T *be a mapping from* X *into* X. *Suppose that there exists* $r \in [0, 1)$ *such that*

$$\varphi(r)d(x, Tx) \leq d(x, y)$$

implies

$$d(Tx, Ty) \leq r \cdot \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\right\}$$

for all $x, y \in X$, *where the function* ϕ *is defined as in Theorem 2.1. Then, there exists* $z \in X$ *such that* $z = Tz$.

Corollary 2.3 is the generalization fixed point theorem [4]. Corollary 2.3 also is the generalization of the Theorem 3.1 of Enjouji et al. [14], since by symmetry, the inequality (3.3) in [14] implies the inequality (1) in Theorem 1.1. Considering generalizations of the Theorem 1.2, Popescu [15] obtained the same result with different function ϕ.

3 An application

The existence and uniqueness of solutions of functional equations and system of functional equations arising in dynamic programming have been studied by using various fixed point theorems (see [12,16,17] and the references therein). In this article, we will prove the existence and uniqueness of a solution for a class of functional equations using Corollary 2.3.

Throughout this section, we assume that U and V are Banach spaces, $W \subset U$, $D \subset V$ and $\mathbb{R}$ is the field of real numbers. Let $B(W)$ denote the set of all the bounded real-valued functions on W. It is well known that $B(W)$ endowed with the metric

$$d_B(h, k) = \sup_{x \in W} |h(x) - k(x)|, \quad h, k \in B(W) \tag{10}$$

is a complete metric space.

According to Bellman and Lee [18], the basic form of the functional equation of dynamic programming is given as

$$p(x) = \sup_{y} H(x, y, p(\tau(x, y))),$$

where x and y represent the state and decision vectors, respectively, $\tau : W \times D \to W$ represents the transformation of the process and $p(x)$ represents the optimal return function with initial state x. In this section, we will study the existence and uniqueness of a solution of the following functional equation:

$$p(x) = \sup_{y}[g(x, y) + G(x, y, p(\tau(x, y))), \quad x \in W \tag{11}$$

where $g : W \times D \to \mathbb{R}$ and $G : W \times D \to \mathbb{R} \to \mathbb{R}$ are bounded functions.

Let a function ϕ be defined as in Theorem 2.1 and the mapping T be defined by

$$T(h(x)) = \sup_{y \in D}\left\{g(x, y) + G(x, y, h(\tau(x, y))\right\}, \quad h \in B(W),\ x \in W. \tag{12}$$

Theorem 3.1. *Suppose that there exists $r \in [0, 1)$ such that for every $(x, y) \in W \times D$, $h, k \in B(W)$ and $t \in W$, the inequality*

$$\varphi(r)d_B(T(h), h) \leq d_B(h, k) \tag{13}$$

implies

$$|G(x, y, h(t)) - G(x, y, k(t))| \leq r \cdot M(h(t), k(t)),$$

where

$$M(h(t), k(t)) = \max\left\{|h(t) - k(t)|,\ |h(t) - T(h(t))|,\ |k(t) - T(k(t))|,\ \frac{|h(t) - T(k(t))| + |k(t) - T(h(t))|}{2}\right\}.$$

Then, the functional equation (11) has a unique bounded solution in $B(W)$.

Proof. Note that T is self-map of $B(W)$ and that $(B(W), d_B)$ is a complete metric space, where d_B is the metric defined by (10). Let λ be an arbitrary positive real number, and $h_1, h_2 \in B(W)$. For $x \in W$, we choose $y_1, y_2 \in D$ so that

$$T(h_1(x)) < g(x, y_1) + G(x, y_1, h_1(\tau_1)) + \lambda, \tag{14}$$

$$T(h_2(x)) < g(x, y_2) + G(x, y_2, h_2(\tau_2)) + \lambda, \tag{15}$$

where $\tau_1 = \tau(x, y_1)$ and $\tau_2 = \tau(x, y_2)$.

From the definition of mapping T and equation (12), we have

$$T(h_1(x)) \geq g(x, y_2) + G(x, y_2, h_1(\tau_2)), \tag{16}$$

$$T(h_2(x)) \geq g(x, y_1) + G(x, y_1, h_2(\tau_1)). \tag{17}$$

If the inequality (13) holds, then from (14) and (17), we obtain

$$\begin{aligned} T(h_1(x)) - T(h_2(x)) &< G(x, y_1, h_1(\tau_1)) - G(x, y_1, h_2(\tau_1)) + \lambda \\ &\leq |G(x, y_1, h_1(\tau_1)) - G(x, y_1, h_2(\tau_1))| + \lambda \\ &\leq r \cdot M(h_1(x), h_2(x)) + \lambda. \end{aligned} \tag{18}$$

Similarly, (15) and (16) imply

$$T(h_2(x)) - T(h_1(x)) \leq r \cdot M(h_1(x), h_2(x)) + \lambda. \tag{19}$$

Hence, from (18) and (19), we have

$$|T(h_1(x)) - T(h_2(x))| \le r \cdot M(h_1(x), h_2(x)) + \lambda. \tag{20}$$

Since the inequality (20) is true for any $x \in W$ and arbitrary $\lambda > 0$, then

$$\varphi(r) d_B(T(h_1), h_1) \le d_B(h_1, h_2)$$

implies

$$d_B(T(h_1), T(h_2)) \le r \cdot \max\left\{d_B(h_1, h_2), d_B(h_1, T(h_1)), d_B(h_2, T(h_2)), \frac{d_B(h_1, T(h_2)) + d_B(h_2, T(h_1))}{2}\right\}.$$

Therefore, all the conditions of Corollary 2.3 are met for the mapping T, and hence the functional equation (11) has a unique bounded solution. □

Authors' contributions
Both authors equitably contributed draft text and the main results section. DĐ contributed the application section. Both authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 14 January 2011 Accepted: 22 August 2011 Published: 22 August 2011

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doi:10.1186/1687-1812-2011-40
Cite this article as: Đorić and Lazović: Some Suzuki-type fixed point theorems for generalized multivalued mappings and applications. *Fixed Point Theory and Applications* 2011 **2011**:40.